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On localised error bounds for orthogonal approximation from shift invariant spaces. (English)

Zbl 0919.42023

Proceedings of the 3rd international conference on functional analysis and approximation theory, Acquafredda di Maratea (Potenza), Italy, September 23–28, 1996. Vols. I and II. Palermo: Circolo Matemàtico di Palermo, Suppl. Rend. Circ. Mat. Palermo, II. Ser. 52, 393-407 (1998).

A shift invariant space $S(N)$ of functions on $\mathbb{R}$ generated by a compactly supported function N whose integer translates are a Riesz basis of $S(N)$ has a unique orthonormal basis generated by a fundamental function of exponential decay. This result is extended by showing how the localization carries over to the error of the orthogonal (i.e., best L_2) approximations $P[f]$ from $S(N)$ to a class of functions f: the paper obtains localized error bounds of the type

$$|f(x) - P_h[f](x)| \le C_{s,p,\rho,\omega} h^{s-\frac{1}{p}} \|\omega^{-1}\rho(h^{-1}x - h^{-1}\cdot)f^{(s)}\|_p,$$

where ω is any function in $L_q(\mathbb{R})$ $(p^{-1} + q^{-1} = 1)$ such that the norm on the right is finite, and ρ is a determined radial function of exponential decay. Optimal choices of ρ are characterized using the zeros of the generalized Euler-Frobenius polynomial. The main tool is provided by the associated Peano kernel whose decay properties are studied.

For the entire collection see [Zbl 0892.00039].

Reviewer: A.L.Brown (Chandigarh)

MSC:

42C15 General harmonic expansions, frames
41A30 Approximation by other special function classes
41A50 Best approximation, Chebyshev systems

Keywords:

shift invariant spaces; reproducing kernel; wavelets; Riesz basis; error bounds; Peano kernel