Trudinger, Neil S.; Wang, Xu-Jia
**Hessian measures. I.** (English) Zbl 0915.35039
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Let $\Omega$ be a domain in Euclidean $n$-space $\mathbb{R}^n$. For $k = 1, \dots, n$ and $u \in C^2(\Omega)$ the $k$-Hessian operator $F_k$ is defined by $F_k[u] = S_k(\lambda(D^2u))$, where $\lambda = (\lambda_1, \dots, \lambda_n)$ denotes the eigenvalues of the Hessian matrix of second derivatives $D^2u$, and $S_k$ is the $k$th elementary symmetric function on $\mathbb{R}^n$, given by

$$S_k(\lambda) = \sum_{i_1 < \cdots < i_k} \lambda_{i_1} \cdots \lambda_{i_k}.$$

Our purpose in this paper is to extend the definition of the $F_k$ to corresponding classes of continuous functions so that $F_k[u]$ is a Borel measure and to consider the Dirichlet problem in this setting. We shall prove that $F_k[u]$ may be extended to the class of $k$-convex functions in $C^0(\Omega)$ as a Borel measure $\mu_k$, for all $k = 1, \dots, n$,, and that the corresponding mapping $u \to \mu_k[u]$ is weakly continuous on $C^0(\Omega)$. The resultant measure $\mu_k[u]$ will be called the $k$-Hessian measure generated by $u$.

**MSC:**

35J60 Nonlinear elliptic equations
28A33 Spaces of measures, convergence of measures
35B05 Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs



**Keywords:**

Hessian operator; $k$-convex functions

**Full Text:** DOI