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Adjoint linear series on weakly 1-complete Kähler manifolds. I: Global projective embedding. (English) Zbl 0912.32021

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A complex manifold X is said to be weakly 1-complete if there exists a smooth function $\Phi : X \to \mathbb{R}$ which is plurisubharmonic and exhaustive. For each point x on X, put $d(x) = \max\{\dim V : V$ is a compact subvariety of X passing through $x\}$.

The main results are the following.

Theorem 1. Let X be an n-dimensional weakly 1-complete manifold with a positive line bundle L. Then $K_X \otimes L^{\otimes m}$ is ample for every $m > n(n+1)/2$.

Theorem 2. Let $x_1, \dots, x_r$ be r distinct points on a sublevel set X_c and let $d_x = \max\{d(x_i) : i = 1, \dots, r\}$. Then for every positive integer $m > \frac{1}{2} d_x(d_x + 2r - 1)$, the restriction map $H^0(X_c, K_X \otimes L^{\otimes m}) \to \oplus_{i=1}^r \mathcal{O}_X/\mathcal{M}_{X,x_i}$ is surjective.

Theorem 3. Let X be a weakly 1-complete manifold. Then the following three statements are equivalent. (1) X is holomorphically embeddable into a projective space. (2) X admits a positive line bundle. (3) There exists an integral Kähler form on X.

Theorem 4. Every holomorphically convex complex manifold with a positive line bundle admits a proper holomorphic embedding into a product space of a projective space and a complex Euclidean space.

Theorem 5. A weakly 1-complete manifold with a negative canonical bundle is Stein if and only if it has no compact subvarieties of positive dimension.

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MSC:

32J25 Transcendental methods of algebraic geometry (complex-analytic aspects)

32H02 Holomorphic mappings, (holomorphic) embeddings and related questions in several complex variables

Keywords:

adjoint linear series; weakly 1-complete manifold; embedding theorem

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