Buttazzo, Giuseppe; Giaquinta, Mariano; Hildebrandt, Stefan

One-dimensional variational problems. An introduction. (English) Zbl 0915.49001

Oxford Lecture Series in Mathematics and its Applications. 15. Oxford: Clarendon Press. viii, 262 p. (1998).

This well written book deals with the one-dimensional variational problems in a modern way. Such problems are connected with ordinary differential equations, so the theory requires less technical prerequisites in comparison with Calculus of Variations for multiple integrals involving PDE's, yet it still provides us with the same kind of suprising phenomena.

In Chapter 1 the classical indirect methods based on necessary and sufficient conditions of optimality are presented. After deriving the Euler equation and other necessary conditions the authors describe the "Carathéodory royal road" to the field theory and they finish this chapter discussing the famous classical examples like Fermat principle, the Newton problem of optimal aerodynamic profile, the brachistochrone or heavy chain problem, the radially symmetric minimal surfaces problem, the elastic string and beam problems and so on. In Chapter 2 the basic function spaces are introduced. The concept of the space $\mathrm{AC}(a, b)$ of absolutely continuous functions defined on an interval $(a, b) \subset \mathbb{R}$, introduced as by Vitali and then used by Tonelli, is related to that of Sobolev spaces $H^{1,p}(a, b)$, $p \geq 1$. A larger class $\mathrm{BV}(a, b)$ of functions with bounded variation is also studied. Chapter 3 is devoted to the study of the lower semicontinuity and existence problems. The special emphasis is put on the direct method which leads to various existence results depending on various topologies introduced in AC, BV and Sobolev spaces. In Chapter 4 some regularity results for minimizers of one-dimensional variational problems are presented. Special attention is paid to the case of regular integrands studied by Hilbert and to the partial regularity theorem of Tonelli. The Lavrentiev phenomenon is also discussed and some examples due to Ball-Mitzel showing that Tonelli's result is optimal are quoted. In Chapter 5 some applications and significant examples are given. For instance, many boundary-value problems and eigen-value problems (e.g., related to Sturm-Liouville operators and vibrating string problem) are treated in detail, as well as the existence of periodic solutions to Hamiltonian systems and to variational problems (also without coercivity assumptions). Some existence and regularity results for obstacle problems related to parametric and non-parametric variational problems are stated and their link with variational inequalities is indicated. An approach to existence problems in optimal control theory for systems described by ODE's is also given. In the "Scholia" added at the end of the book one can find several interesting historical remarks on the calculus of variations. This chapter can also be considered as a guide to the literature containing information on basic concepts and recent developments in the field.

Reviewer: Z.Denkowski (Kraków)

MSC:

49-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to calculus of variations and optimal control
49J05	Existence theories for free problems in one independent variable
49K05	Optimality conditions for free problems in one independent variable
49L99	Hamilton-Jacobi theories
34B24	Sturm-Liouville theory

Keywords:

indirect and direct methods; compactness and lower semicontinuity; $\mathrm{AC}(a, b)$; $\mathrm{BV}(a, b)$; Sobolev spaces $H^{1,p}(a, b)$