Viola, Carlo
Hypergeometric functions and irrationality measures. (English) ⟨Zbl 0904.11020⟩
Motohashi, Y. (ed.), Analytic number theory. Proceedings of the 39th Taniguchi international symposium on mathematics, Kyoto, Japan, May 13–17, 1996. Cambridge: Cambridge University Press. Lond. Math. Soc. Lect. Note Ser. 247, 353-360 (1997).

The author shows how the analogue for one-dimensional Euler-Pochhammer integrals of the method developed in [*G. Rhin* and *C. Viola*, Acta Arith. 77, No. 1, 23-56 (1996; Zbl 0864.11037)] can be applied to obtain very easily good irrationality measures for the value of the logarithm at rational points. Let h, j, l be integers satisfying $h > \max\{0, -l\}$, $j > \max\{0, l\}$, and define $M = \max\{j - l, h + l\}$. By applying the Euler-Pochhammer integral representation

$$_2F_1(\alpha, \beta; \gamma; y) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma - \beta)} \int_0^1 \frac{x^{\beta-1}(1 - x)^{\gamma-\beta-1}}{(1 - xy)^\alpha} dx$$

of the Gauss hypergeometric function, one obtains the least irrationality measure

$$\mu(\log(1 + r/s)) \leq \frac{U}{V}$$

for integers r and s satisfying $r \neq 0$, $s \geq 1$, $r > -s$, $(r, s) = 1$, provided that $V > 0$. Here,

$$U = \log|f(x_1)| - \log f(x_0)$$

and

$$V = -\log f(x_0) + \int_\Omega d\psi(x) - M(1 + \log s) - (h + j)\log|r/s| + \min\{0, l\}\log(1 + r/s),$$

where Ω is the set of $\omega \in [0, 1)$ satisfying $[(j - l)\omega] + [(h + l)\omega] < [h\omega] + [j\omega]$, $\psi(x) = \Gamma'(x)/\Gamma(x)$, x_0 and x_1 are the stationary points $\neq 0, 1$ of the function

$$f(x) = \frac{x^h(1 - x)^j}{(1 + (r/s)x)^{j-l}}$$

with $0 < x_0 < 1$ and $1 + (r/s)x_1 < 0$. The method yields the best known irrationality measures of a class of logarithms of rational numbers. Especially, a simple proof of the best known irrationality measure of $\log 2$, 3.89139978 [*E. A. Rukhadze*, Mosc. Univ. Math. Bull. 42, No. 6, 30-35 (1987); translation from Vestn. Mosk. Univ., Ser. I 1987, No. 6, 25-29 (1987; Zbl 0635.10025)], can be obtained.

For the entire collection see [Zbl 0874.00035].

Reviewer: Takao Komatsu (Tsu)

MSC:

11J91	Transcendence theory of other special functions
11J82	Measures of irrationality and of transcendence
33C05	Classical hypergeometric functions, $_2F_1$

Cited in **1** Review
Cited in **5** Documents

Keywords:

irrationality measures; Gauss hypergeometric functions; Euler-Pochhammer integrals; logarithm at rational points