Apel, Thomas; **Nicaise, Serge**

The finite element method with anisotropic mesh grading for elliptic problems in domains with corners and edges. (English) Zbl 0911.65107

Math. Methods Appl. Sci. 21, No. 6, 519-549 (1998).

The authors are interested in the Dirichlet problem for second-order elliptic equations in non-convex three-dimensional polygonal domains. The equations have constant coefficients. Such problems can exhibit singularities along edges and at corners. Unless properly treated, these singularities will diminish the rate of convergence of finite element solutions. In the current paper, the authors devise a mesh refinement strategy based on singularity exponents that results in optimal convergence rates when used with piecewise linear shape functions. The mesh refinement is anisotropic in the sense that the aspect ratio of the mesh is not necessarily bounded. The authors present some new local interpolation error estimates employing anisotropic weighting functions to support their convergence results.

The first-named author and his colleagues have investigated similar problems before. The present paper is the first to employ the anisotropic mesh refinement strategy for problems with both edge and corner singularities in arbitrary polygonal domains. A numerical example shows that optimal convergence can be achieved in practice and is a substantial improvement over uniform mesh refinement for a problem with a so-called Fichera corner.

Reviewer: Myron Sussman (Bethel Park)

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N50 Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs
65N12 Stability and convergence of numerical methods for boundary value problems involving PDEs
35J25 Boundary value problems for second-order elliptic equations

Keywords:

corner and edge singularities; optimal convergence rate; Dirichlet problem; second-order elliptic equations; mesh refinement; error estimates; numerical example

Full Text: DOI

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