Thomas, Robin

An update on the four-color theorem. (English) Zbl 0908.05040

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K. Appel and *W. Haken* [Bull. Am. Math. Soc. 82, 711-712 (1976; Zbl 0331.05106)] announced a proof of the four-color theorem in 1976, providing an expanded version of that proof in 1989 [Every planar map is four colorable, Contemporary Mathematics 98, American Mathematical Society (1989; Zbl 0681.05027)]. In 1997, *N. Robertson*, *D. Sanders*, *P. Seymour* and *R. Thomas* published a greatly simplified proof; see [J. Comb. Theory, Ser. B 70, No. 1, 2-44 (1997; Zbl 0883.05056)]. In the present paper, the fourth of the latter authors summarizes the history of the theorem, gives several equivalent formulations (in a surprising variety of contexts, including vector cross products in $\mathbb{R}^3$, Lie algebras, and divisibility by 7), and discusses aspects of the new proof-including progress on some generalizations. To understand two recent results in this connection by *N. Robertson*, *P. Seymour* and *R. Thomas* [J. Comb. Theory, Ser. B 70, No. 1, 166-183 (1997; Zbl 0883.05055) and Combinatorica 13, No. 3, 279-361 (1993; Zbl 0830.05028)], define a graph G to be apex if $G \setminus v$ is planar for some v in $V(G)$ and doublecross if G can be drawn in the plane with two crossings, both in the same region. Theorem 1. Let G be a counterexample of minimum order to the conjecture that every cubic graph with no cut-edge and no edge 3-coloring has a Petersen minor; then G is apex or doublecross. (The conjecture implies the four-color theorem.) Theorem 2. Let G be a counterexample of minimum order to Hadwiger's conjecture that if a graph has no K_6 minor, then it has a 5-coloring (another generalization of the four-color theorem); then G is apex.

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MSC:

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