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Growth of entire and meromorphic functions. (English. Russian original) Zbl 0942.30019

Sb. Math. 189, No. 6, 875-899 (1998); translation from Mat. Sb. 189, No. 6, 59-84 (1998).

For a function f meromorphic in the plane let $b(\infty, f) = \liminf_{r\to\infty} \log M(r, f)/A(r, f)$, where $A(r, f) = rT'(r, f)$. Here $T(r, f)$ is the Ahlfors-Shimizu characteristic. It was shown by *H. Bock* and the reviewer [J. Anal. Math. 64, 327-336 (1994; Zbl 0828.30013)] that if the order λ of f satisfies $\frac{1}{2} \le \lambda \le \infty$, then $b(\infty, f) \le \pi$. The author improves this result for functions attaining the maximum modulus at more than one point. Let $p(r, \infty, f)$ be the number of components of the set $\{\varphi : |f(re^{i\varphi})| > 1\}$ that contain a point ϕ such that $M(r, f) = |f(re^{i\phi})|$. Let $p(\infty, f) = \liminf_{r\to\infty} p(r, \infty, f)$. It is shown (Theorem 1) that if $\frac{1}{2} \le \lambda/p(\infty, f) \le \infty$, then $b(\infty, f) \le \pi/p(\infty, f)$. The sharp upper bound for $b(\infty, f)$ is also given for the case $\lambda/p(\infty, f) < \frac{1}{2}$. For $a \in \mathbb{C}$ let $b(a, f) = b(\infty, 1/(f - a))$ and let $\Delta := \Delta(a, f)$ be the Valiron deficiency of a. It is shown (Theorem 2) that $b(a, f) \le \pi\sqrt{\Delta(2-\Delta)}$ if $\frac{1}{2} \le \lambda \le \infty$ or if $0 < \lambda < \frac{1}{2}$ and $\sin \pi\lambda/2 \ge \sqrt{\Delta/2}$. The upper bound for $b(a, f)$ for the remaining case is also given. *A. Eremenko* [Complex Variables, Theory Appl. 34, No. 1-2, 83-97 (1997; Zbl 0905.30025)] has shown that if $\frac{1}{2} \le \lambda \le \infty$, then $\sum_{a\in\overline{\mathbb{C}}} b(a, f) \le 2\pi$. Here it is shown for $\Delta := \Delta(0, f')$ that $\sum_{a\in\overline{\mathbb{C}}} b(a, f) \le 2\pi\sqrt{\Delta(2-\Delta)}$ for meromorphic f (Theorem 3) and $\sum_{a\in\mathbb{C}} b(a, f) \le \pi\sqrt{\Delta(2-\Delta)}$ for entire f (Theorem 4). The author points out that his method of proof is different from that of Eremenko. Finally the author proves two results relating $b(\infty, f)$ and $p(\infty, f)$ to the spread

$$\sigma(\infty, f) = \limsup_{r\to\infty} \operatorname{meas}\{\varphi : |f(re^{i\varphi})| > 1\}.$$

Results similar to those obtained in this paper, with $b(\infty, f)$ replaced by the Petrenko deviation $\beta(\infty, f) = \liminf_{\to\infty} \log M(r, f)/T(r, f)$, were obtained by the author and *A. I. Shcherba* in previous papers [see Mat. Sb. 181, No. 1, 3-24 (1990; Zbl 0716.30024) and Mat. Sb. 186, No. 3, 85-102 (1995; Zbl 0854.30023)]. The methods developed there, which in part are based on Baernstein's star function, are extended and modified in order to obtain the results of the present paper.

Reviewer: W.Bergweiler (Kiel)

MSC:

30D35	Value distribution of meromorphic functions of one complex variable, Nevanlinna theory
30D30	Meromorphic functions of one complex variable, general theory
30D20	Entire functions of one complex variable, general theory

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