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Homomorphisms of Barsotti-Tate groups and crystals in positive characteristic. – Erratum. (English) Zbl 0929.14029

Invent. Math. 134, No. 2, 301-333 (1998); erratum ibid. 138, No. 1, 225 (1999).

Let R be a discrete valuation ring, K its fraction field, and G and H p-divisible groups over R. In the case of char$K = 0$, *J. T. Tate* [in: "p-divisible groups" Proc. Conf. local Fields, NUFFIC Summer School Driebergen 1966, 158-183 (1967; Zbl 0157.27601)] proved that the natural map $\mathrm{Hom}_R(G, H) \to \mathrm{Hom}_K(G_K, H_K)$ is a bijection.

In the present paper the author obtains the analogous result in positive characteristic as a consequence of a general result on F-crystals.

More precisely, if one considers the natural inclusion $j : \eta \to S$, where $S = \mathrm{Spec}R$ and $\eta = \mathrm{Spec}K$, then the author proves theorem 1.1:

If char$K = p > 0$ and R has a p-basis, then the natural functor

$$j^* : \{\text{non-degenerate} F\text{-crystals}/S\} \to \{\text{non-degenerate} F\text{-crystals}/\eta\}$$

is fully faithful.

From general facts of Dieudonné crystalline theory [cf. *P. Berthelot* and *W. Messing*, in: "The Grothendieck Festschrift", Collect. Artic. in Honor of the 60th Birthday of A. Grothendieck. Vol. I, Prog. Math. 86, 173-247 (1990; Zbl 0753.14041)], the author deduces as a corollary that the map $\mathrm{Hom}_R(G, H) \to \mathrm{Hom}_K(G_K, H_K)$ is bijective, without assumptions on R. As applications of this corollary, in section 2, the author proves in this situation two results about the relations among abelian varieties and their p-divisible groups.

Precisely he proves a criterion for good reduction: Let X_η be an abelian variety over η with p-divisible group G_η. Then X_η has good reduction if and only if G_η has good reduction. The same holds for semi-stable reduction.

The second result is the following theorem. Let F be a field finitely generated over $\mathbb{F}_p$. Let X and Y be abelian varieties over F and denote by $X[p^\infty]$ and $Y[p^\infty]$ their p-divisible groups. Then there is an isomorphism $\mathrm{Hom}(X, Y) \otimes \mathbb{Z}_p \cong \mathrm{Hom}(X[p^\infty], Y[p^\infty])$.

The sections 3-9 of the paper are devoted to the proof of theorem 1.1.

In the erratum [Invent. Math. 138, No. 1, 225 (1999)], the author points out a minor mistake in lemma 2.1, part III.

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MSC:

14L05	Formal groups, p-divisible groups
14F30	p-adic cohomology, crystalline cohomology
14G20	Local ground fields in algebraic geometry

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Barsotti-Tate groups; crystals; Dieudonné crystalline theory; p-divisible groups; abelian variety; semi-stable reduction

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