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Flattening antichains with respect to the volume. (English) Zbl 0913.05092

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Given a (finite) poset $(P, \leq)$ with a strictly increasing function $h : P \to N = \{0, 1, 2, \dots\}$, define the volume of a subset S of P (with respect to h) to be $\sum_{x \in S} h(x)$. A subset S of P is flat (with respect to h) if $x, y \in S$ with $h(x) < h(y)$ implies there is no $z \in P$ such that $h(x) < h(z) < h(y)$. Thus, if $P = B_n$, the lattice of subsets of $[n] = \{1, \dots, n\}$, and $h(x) = |x|$, then S is flat if there exists a k such that every element of S has cardinality k or $k + 1$. There are many posets which are themselves flat with respect to some function h. Many more (e.g., chains) have flat antichains only. It has been conjectured that B_n is among the posets which have the property that for every antichain there is a flat antichain with the same volume ($h(x) = |x|$). Special cases occur in the literature. In this paper the conjecture is settled positively via a sequence of quite elegant technical lemmas based on results by Sperner, Clements and Daykin, among others. Rather than closing a area of inquiry it may also be the case that this paper can be adapted to a more general situation and lead to other interesting conjectures.

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MSC:

05D99 Extremal combinatorics
06A07 Combinatorics of partially ordered sets
06E99 Boolean algebras (Boolean rings)

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