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Extremal problems on Pompeiu sets. (English. Russian original) Zbl 0957.53042

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A compact subset K of $\mathbb{R}^n$, $n \geq 2$, is said to have the Pompeiu property if, whenever a function $f \in C(\mathbb{R}^n)$ averages to zero on every set $\sigma(k)$, $\sigma \in M(n)$, the group of rigid motions of $\mathbb{R}^n$, then $f \equiv 0$. Since no ball has the Pompeiu property, one considers the same question for pairs B_1, B_2 of closed balls of different radii r_1, r_2, respectively. It is known that there is a countable exceptional set P_n such that the pair B_1, B_2 will have the Pompeiu property if and only if $r_1/r_2 \notin P_n$. We refer to [*L. Zalcman*, Am. Math. Monthly 87, 161-175 (1980; Zbl 0433.53048)] for an introduction to this kind of problem and to the bibliography compiled by *L. Zalcman* in [NATO ASI Ser., Ser. C 365 185-194 (1992; Zbl 0830.26005), updates available from the author]. Typically this type of problem can be reduced to a problem of harmonic analysis of ideals in the space of distributions of compact support in $\mathbb{R}^n$. This depends in an essential way on the fact that one is posing the problem in the whole space. In [J. Anal. Math. 52, 133-166 (1989; Zbl 0668.30037)], *R. Gay* and this reviewer showed that one could prove local versions of this theorem, that is, $\mathbb{R}^n$ replaced by a ball B. Clearly, there is an obvious minimum size condition on B, namely that the radius of r of B is sufficiently large so that K or a translate of K, fits into B. It turns out for the case of two balls B_1, B_2 of respective radii r_1, r_2, one needs that $r_1 + r_2 < r$. There are similar theorems for other K. In the present paper the author gives sharp bounds for this type of size conditions.

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