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The Fujita conjecture and the extension theorem of Ohsawa-Takegoshi. (English)

Zbl 0941.32021
Noguchi, J. (ed.) et al., Geometric complex analysis. Proceedings of the conference held at the 3rd International Research Institute of the Mathematical Society of Japan, Hayama, March 19-29, 1995. Singapore: World Scientific. 577-592 (1996).

From the introduction: "Let L be an ample line bundle over a compact complex manifold X of complex dimension n. We discuss here the most recent result of myself and *U. Angehrn* [Invent. Math. 122, No. 2, 291-308 (1995; Zbl 0847.32035)] on the conjecture of Fujita on freeness. Fujita's conjecture states that $(n+1)L+K_X$ is free. The conjecture of Fujita has a second part on very ampleness which states that $(n+2)L+K_X$ is very ample. We will confine ourselves to the freeness part of the Fujita conjecture. My result with Angehrn is the following.

Main Theorem. Let κ be a positive number. If $(L^d\cdot W)^{\frac{1}{d}}\geq \frac{1}{2}n(n+2r_1)+\kappa$ for any irreducible subvariety W of dimension $1\leq d\leq n$ in X, then the global holomorphic sections of $L+K_X$ over X separate any set of f distinct points $P_1,\dots,P_r$ of X. In other words, the restriction map $\Gamma(X,L)\to\oplus_{\nu=1}^r\mathcal{O}_X/\mathfrak{m}_{P_\nu}$ is surjective, where $\mathfrak{m}_{P_\nu}$ is the maximum ideal at P_ν.

Corollary. $mL+K_X$ is free for $m\geq\frac{1}{2}(n^2+n+2)$".

For the entire collection see [Zbl 0903.00037].

MSC:

32L10 Sheaves and cohomology of sections of holomorphic vector bundles, general results
14C20 Divisors, linear systems, invertible sheaves
32J25 Transcendental methods of algebraic geometry (complex-analytic aspects)

Keywords:

extension theorem of Ohsawa-Takegoshi; conjecture of Fujita; freeness; holomorphic sections