Erdős, Paul; **Gyárfás, András**

A variant of the classical Ramsey problem. (English) Zbl 0910.05034
Combinatorica 17, No. 4, 459-467 (1997).

The following quantity is estimated. Let $f(n,p,q)$ be the minimum number of colors needed to color all edges of K_n such that every K_p gets at least q colors. A general upper bound is given using the Lovász local lemma. If $q=\binom{p}{2}-p+3$ then $f(n,p,q)$ is linear while $f(n,p,q-1)$ is sublinear. If $q=\binom{p}{2}-\lfloor\frac{p}{2}\rfloor+2$ then $f(n,p,q)=\Omega(n^2)$ while $f(n,p,q-1)=O(n^{2-\frac{4}{p}})$ but is $\Omega(n^{\frac{4}{3}})$ for $p\geq 7$. $f(n,p,p)=\Omega(n^{\frac{1}{p-2}})$. Also, $\frac{5}{6}(n-1)\leq f(n,4,5)$ and $f(n,9,34)=\binom{n}{2}-o(n^2)$.

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MSC:

05C35 Extremal problems in graph theory
05C80 Random graphs (graph-theoretic aspects)
05D10 Ramsey theory
05C55 Generalized Ramsey theory

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References:

[1] W. G. Brown, P. Erdos, V. T. Sós: Some extremal problems onr-graphs, inNew directions in the theory of Graphs, Proc. 3rd Ann Arbor Conference on Graph Theory, 53–63, Academic Press, New York, 1973.

[2] W. G. Brown, P. Erdos, V. T. Sós: On the existence of triangulated spheres in 3-graphs and related problems,Periodica Mathematica Hungarica,3 (1973), 221–228. · Zbl 0269.05111 · doi:10.1007/BF02018585

[3] P. Erdos: Solved and unsolved problems in combinatorics and combinatorial number theory,Congressus Numerantium,32 (1981), 49–62.

[4] P. Erdos: Extremal problems in graph theory, inTheory of Graphs and its Applications (M. Fiedler, ed.), Academic Press, New York, 1964, 29–36.

[5] P. Erdos, D. J. Kleitman: On coloring graphs to maximize the proportion of multicoloredk-edges,J. of Combinatorial Theory,5 (1968), 164–169. · Zbl 0167.22302 · doi:10.1016/S0021-9800(68)80051-1

[6] P. Erdos, L. Lovász: Problems and results on 3-chromatic hypergraphs and some related problems, in:Infinite and Finite sets, Colloquia Math. Soc. J. Bolyai vol. 10 (A. Hajnal et al eds.) North-Holland, Amsterdam, 609–617 1995.

[7] P. Erdos, V. T. Sós: personal communication, 1994.

[8] P. C. Fishburn: personal communication, 1995.

[9] A. Gyárfás, J. Lehel: Linear Sets with Five Distinct Differences among any Four Elements,J. of Combinatorial Theory B,64 (1995), 108–118. · Zbl 0830.05061 · doi:10.1006/jctb.1995.1028

[10] I. Z. Ruzsa, E. Szemerédi: Triple systems with no six points carrying three triangles, inCombinatorics (Keszthely, 1976), Coll. Math. Soc. J. Bolyai 18, Volume II. 939–945.

[11] W. D. Wallis: Combinatorial Designs, Monographs and Textbooks inPure and Applied Mathematics, Vol. 118, Marcel Dekker, Inc. 1988.