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Calculation of Fuchsian groups associated to billiards in a rational triangle. (English)

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Summary: We define, following Veech, the Fuchsian group $\Gamma(P)$ of a rational polygon P. If P is simply-connected, then 'rational' is equivalent to the condition that all interior angles of P be rational multiples of π. Should it happen that $\Gamma(P)$ has finite covolume in $\mathrm{PSL}(2,\mathbb{R})$ (and is thus a lattice), then a theorem of Veech states that every billiard path in P is either finite or uniformly distributed in P.

We consider the Fuchsian groups of various rational triangles. First, we calculate explicitly the Fuchsian groups of a new sequence of triangles, and discover they are lattices. Interestingly, the lattices found are not commensurable with those previously known. We then demonstrate a class of triangles whose Fuchsian groups are not lattices. These are the first examples of such triangles. Finally, we end by showing how one may specify algebraically, i.e., by an explicit polynomial in two variables, the Riemann surfaces and holomorphic one-forms that are associated to a simply-connected rational polygon. Previously, these surfaces were known by their geometric description. As an example, we show a connection between the billiard in a regular polygon and the well-known Fermat curves of the algebraic equation $x^n + y^n = 1$.

MSC:

37A99 Ergodic theory
37G05 Normal forms for dynamical systems

Keywords:

Fuchsian group $\Gamma(P)$; Veech states; lattices; Riemann surfaces; holomorphic 1-forms; billard; Fermat curves

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