Kinoshita, Tamotu

On the wellposedness in the Gevrey classes of the Cauchy problem for weakly hyperbolic systems with Hölder continuous coefficients in time. (English) Zbl 0933.35119

Osaka J. Math. 35, No. 4, 735-750 (1998).

The author considers systems of the form

$$\partial_t u = \sum A_h(t)\partial_h u + B(t)u \tag{*}$$

on $[0,T]\times\mathbb{R}^n_\eta$ with the initial condition $u(0,x)=u_0(x)$, where $A_h(t)$, $B(t)$ are $N\times N$ matrices, A_h is Hölder continuous, B is continuous, and the system (*) is weakly hyperbolic. Two cases are considered: First, when no condition is imposed (besides the weak hyperbolicity) and second, when there exists a non-singular matrix $P(t,\xi)$ such that

$$P(t,\xi)A(t,\xi)P(t,\xi)^{-1} = \mathrm{diag}\{D_1,D_2,\dots,D_k\} \quad \text{for some } 1\le k\le N$$

and the D_j are triangular matrices of size $m_j\times m_j$, whose diagonal elements are real, and moreover

$$\left|P(t,\xi)\right| + \left|P(t,\xi)\right|^{-1} \le C \text{ for any } t\in[0,T], \quad |\xi|=1.$$

The result proved by the author, which generalizes previous results of *E. Jannelli* [Ann. Mat. Pura Appl., IV. Ser. 140, 133-145 (1985; Zbl 0583.35074)], is the following: Let $0<p_0<\infty$ and $\nu_0>0$. Then there exists $\nu>0$ such that for any $u_0\in L^2_{\rho,k,\nu_0}(\mathbb{R}^n)$ the Cauchy problem has an unique solution $u\in C^1([0,T],$ $L^2_{\rho_1,k,\nu}(\mathbb{R}^n))$ provided $0<p_1<\rho_0$, $1<s<\frac{\mu(1+\sigma^{-1})}{\mu(1+\sigma^{-1})-1}$ where $\mu=N$ (case 1) or $\mu=\max_{1\le i\le k} m_i$ (case 2) and $s=\frac{1}{k}$. Here $L^2_{p,k,\nu}(\mathbb{R}^n)=\{u\in L^2(\mathbb{R}^n),\ e^{\rho\langle\xi\rangle^k_\nu}\widehat{\mu}\in L^2(\mathbb{R}^n)\}$, where $\langle\xi\rangle_\nu=(|\xi|^2+\nu^2)^{1/2}$. The proof uses energy estimates and some nontrivial algebraic lemmas. For the existence part the author considers an associated system of the form

$$\partial_t u_l = \sum A_h(t) il\sin(Dh/l)u_l + B(t)u_l \tag{**}$$

and since $il\sin(Dh/l)$ belong to OPS° for any fixed l the right hand side of (**) is a bounded linear operator in $L^2_{p_1,k,\nu}(\mathbb{R}^n)$, which ensures the existence and unicity of u_l. Moreover $\{e^{\rho_1\langle\xi_l(D)\rangle^k_\nu}u_l\}$ is bounded on L^2, and its weak limit is a solution of (*), with the required regularity.

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MSC:

35L40	First-order hyperbolic systems	Cited in **1** Document
35A05	General existence and uniqueness theorems (PDE) (MSC2000)	

Keywords:

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