Nazarov, S. A.

Justification of the asymptotic theory of thin rods. Integral and pointwise estimates. (Russian) Zbl 0917.35029

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The author studies the following system of elliptic equations which comes from the modeling of a thin rod in the frameworks of linear elasticity:

$$\begin{aligned} D(-\nabla_x)AD(\nabla_x)^t u = f, &\quad \text{in } \Omega_h, \\ D(n)AD(\nabla_x)^t = g, &\quad \text{on } \Gamma_h = \{x \in \partial\Omega_h : |z| < 1\}, \\ u = 0, &\quad \text{on } \gamma_h^{\pm} = \{x \in \partial\Omega_h : z = \pm 1\}, \end{aligned}$$

where Ω_h denotes a thin rod located in the strip $\{x = (y, z) : y \in \mathbb{R}^2,\ z \in (0, 1)\}$ and is given by the equality

$$\Omega_h = \{x : |z| < 1,\ \xi = (\eta, \zeta) \equiv h^{-1}(y, z) \in Q \subset \mathbb{R}^3\};$$

here $h = 1/N$ is a small parameter, ξ is the "fast" variable. The rod is assumed to be nonhomogeneous and the end-walls $\gamma_h^{\pm}$ of the rod Ω_h are rigidly fixed, which is expressed by the last condition in the model. The aim of the article is to construct and justify the asymptotic of a solution to the above problem as $h \to +0$. This allows us to expose an asymptotic deduction of the one-dimensional equations of the theory of thin rods and then to estimate the error which arises due to reduction of dimension. The author observes that the passage to the "slow" variable z is also possible.

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MSC:

35J45 Systems of elliptic equations, general (MSC2000)
35B45 A priori estimates in context of PDEs
74B05 Classical linear elasticity
35B40 Asymptotic behavior of solutions to PDEs
35C20 Asymptotic expansions of solutions to PDEs

Keywords:

elasticity equations; integral and pointwise estimates