

Major, P.

Almost sure functional limit theorems. I: The general case. (English) Zbl 0921.60033
Stud. Sci. Math. Hung. 34, No. 1-3, 273-304 (1998).

The aim of this interesting paper is to obtain the almost sure functional limit theorem (ASFLT) for arbitrary self-similar process $X(t, \omega)$. This theorem states that for every bounded measurable functional \mathcal{F} on the space $D[0, 1]$ (or $C[0, 1]$ if X is continuous)

$$(\log T)^{-1} \int_1^T \mathcal{F}(X_t(\cdot, \omega)) t^{-1} dt \rightarrow E\mathcal{F}(X(\cdot, \omega)) \quad \text{as } T \rightarrow \infty$$

holds for almost all ω , where $X_t(u, \omega) = t^{-1/\alpha} X(ut, \omega)$, $0 \leq u \leq 1$, $t > 0$, $\alpha > 0$. The main idea is that ASFLT for $X(t, \omega)$ follows in a natural way from the ergodic theorem for the generalized Ornstein-Uhlenbeck process corresponding to $X(t, \omega)$. Next it is shown by an appropriate coupling argument that ASFLT holds also for sequences of random broken lines (or polygons) determined by certain sequences of random variables. Thus the almost sure central limit theorems appearing in the literature of the last decade are special corollaries of the results of this paper.

Reviewer: [Tadeusz Inglot \(Wrocław\)](#)

MSC:

- 60F17 Functional limit theorems; invariance principles
- 60F15 Strong limit theorems
- 60G18 Self-similar stochastic processes
- 28D05 Measure-preserving transformations

Cited in **1** Review
Cited in **3** Documents

Keywords:

almost sure invariance principle; ergodic theorem; self-similar process; Ornstein-Uhlenbeck process