

Máté, László

On infinite composition of affine mappings. (English) Zbl 0939.47006
Fundam. Math. 159, No. 1, 85-90 (1999).

Let \mathcal{J} be the set of infinite sequences of the symbols $1, \dots, N$ i.e. $\mathcal{J} = \{1, \dots, N\}^\infty$ and let s be the shift operator on \mathcal{J} which is defined for each $\sigma = (\sigma_1, \dots, \sigma_n, \dots) \in \mathcal{J}$ as follows

$$s(\sigma_1, \dots, \sigma_n, \dots) = (\sigma_2, \sigma_3, \dots, \sigma_{n+1}, \dots); \quad \sigma_i \in \{1, \dots, N\}, \quad i \in \mathbb{N}.$$

The dynamical system $[\mathcal{J}, s]$ thus obtained with the usual metric

$$d_c[\omega, \sigma] = \sum_{i=1}^{\infty} \frac{|\omega_i - \sigma_i|}{N^i}; \quad \omega, \sigma \in \mathcal{J},$$

is called the full N -shift. A closed shift-invariant subspace \mathcal{K} of the full N -shift is called a subshift. Denote $B(\mathcal{K}) = \{(\sigma_i, \dots, \sigma_j)_{j \geq i} : \sigma = (\sigma_1, \dots, \sigma_n, \dots) \in \mathcal{K}\}$. Let us suppose that a positive functional Φ is defined on $B(\mathcal{K})$ such that Φ is submultiplicative i.e. the inequalities

$$\Phi(\sigma_1, \dots, \sigma_n) \leq \Phi(\sigma_1, \dots, \sigma_j) \Phi(\sigma_{j+1}, \dots, \sigma_n), \quad 1 \leq j < n.$$

hold. Define a number $\Phi^* = \Phi^*(K, \Phi)$ as follows

$$\Phi^* = \lim_{n \rightarrow \infty} (\Phi_n^*)^{1/n} \quad \text{where } \Phi_n^* = \max\{\Phi(\sigma_1, \dots, \sigma_n) : \sigma = (\sigma_1, \dots, \sigma_n, \dots) \in \mathcal{K}\}.$$

The main result of the paper is the following

Theorem 3. Let $\{F_{\sigma_i} : i = 1, \dots, N\}$ be affine mappings of \mathbb{R}^n and \mathcal{K} be a subshift of the full N -shift \mathcal{J} . Then the sequence $\{F_{\sigma_1} \circ \dots \circ F_{\sigma_n}\}$ is convergent for every $\sigma \in \mathcal{K}$ and $z \in \mathbb{R}^n$ to a limit $x(\sigma)$ independent of z if and only if $\Phi^* < 1$.

The proof is based on a generalization of the well-known König lemma.

Reviewer: [Aleksandr A. Mekler \(Dresden\)](#)

MSC:

[47A35](#) Ergodic theory of linear operators
[28A80](#) Fractals
[26A18](#) Iteration of real functions in one variable

Cited in **1** Review
Cited in **1** Document

Keywords:

shift invariante subspaces; König lemma; submultiplicative; affine mappings

Full Text: [EuDML](#)