Zhegalov, V. I.; Kotukhov, M. P.

On integral equations for the Riemann function. (English. Russian original) Zbl 0932.35043

Russ. Math. 42, No. 1, 24-28 (1998); translation from Izv. Vyssh. Uchebn. Zaved., Mat. 1998, No. 1, 26-30 (1998).

The paper is devoted to the determination of the Riemann function of some equations. For the equation

$$u_{xy} + au_x + bu_y + cu = 0, \tag{1}$$

the Riemann function is a solution of the conjugated equation $v_{xy} - (av)_x - (bv)_y + cv = 0$, satisfying the conditions $v(x,\tau,t,\tau) = \exp\int_t^x b(\xi,\tau)d\xi$, $v(t,y,t,\tau) = \exp\int_\tau^y a(\tau,\eta)d\eta$, where $a, b, c \in \mathbb{C}^2$ in the domain under consideration, and there exist $a_x, b_x \in \mathbb{C}^2$. It is proved that in the case when $a = a_1(y) + \lambda x$, $b = b_1(x) + \lambda y$, $\lambda = \text{const}$, $c - ab - \lambda = \varphi(x)\psi(y)$, the Riemann function of the equation (1) is given by the formula

$$v = J_0\left\{2\left[\int_t^x \varphi(\xi)d\xi \int_\tau^y \psi(\eta)d\eta\right]^{1/2}\right\}\exp\left[\int_t^x b_1(\xi)d\xi + \int_\tau^y a_1(\eta)d\eta + \lambda(xy - t\tau)\right].$$

For the equation

$$u_{xyz} + au_{xy} + bu_{yz} + cu_{xz} + du_x + eu_y + fu_z + gu = 0, \tag{2}$$

the Riemann function is a solution of the equation $v_{xyz} - (av)_{xy} - (bv)_{yz} - (cv)_{xz} + (dv)_x + (ev)_y + (fv)_z - gv = 0$, satisfying the conditions

$$v(t,\tau,z,t,\tau,\vartheta) = \exp\int_\vartheta^z a(t,\tau,\zeta)d\zeta,\ v(t,y,\vartheta,t,\tau,\vartheta) = \exp\int_\tau^y c(t,\eta,\vartheta)d\eta,$$
$$v(x,\tau,\vartheta,t,\tau,\vartheta) = \exp\int_t^x b(\xi,\tau,\vartheta)d\xi.$$

The following expressions are considered:

$$h_1 = a_x + ab - e,\ h_2 = a_y + ac - d,\ h_3 = b_y + bc - f,\ h_4 = b_z + ab - e,$$
$$h_5 = c_x + bc - f, h_6 = c_z + ac - d, h_7 = d_x + bd - g, h_8 = e_y + ce - g, h_9 = f_z + af - g.$$

It is proved that the Riemann function of the equation (2) is given by

$$v(x,y,z,t,\tau,\vartheta) = J_0(2\sqrt{\omega})\exp\sigma(x,y,z,t,\tau,\vartheta)$$

if $h_4 \equiv h_6 \equiv h_9 \equiv 0$, $c = c_1(y,z) + \alpha(z)x$, $b = b_1(x,z) + \alpha(y)y$, $f - bc - \alpha = \varphi(x,z)\psi(y,z)$, where $\omega = \int_t^x \varphi(\xi,\vartheta)d\xi \int_\tau^y \psi(\eta,\vartheta)d\eta$,

$$\sigma = \int_t^x b_1(\xi,\vartheta)d\xi + \int_\tau^y c_1(\eta,\vartheta)d\eta + \int_\vartheta^x a(x,y,\zeta)d\zeta + (xy - t\tau)\alpha(z).$$

Some similar cases are considered as well.

Reviewer: Elena Gavrilova (Sofia)

MSC:

35G05 Linear higher-order PDEs
35C15 Integral representations of solutions to PDEs

Keywords:

Riemann function; integral equations for the Riemann function; conjugated equation