Pascasio, Arlene A.

Tight graphs and their primitive idempotents. (English) Zbl 0927.05085

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Let Γ be a distance-regular graph with diameter $d \geq 3$ and eigenvalues $\theta_0 > \theta_1 > \cdots > \theta_d$. Then

$$\left(\theta_1 + \frac{k}{a_1 + 1}\right)\left(\theta_d + \frac{k}{a_1 + 1}\right) \geq \frac{-ka_1b_1}{(a_1 + 1)^2}.$$

Γ is said to be tight whenever Γ is not bipartite and equality holds above. Suppose E and F are nontrivial primitive idempotents of Γ and the entry-wise product $E \circ F$ is a scalar multiple of a primitive idempotent H of Γ. Then Γ is either bipartite and at least one of E, F is equal to E_d or tight and $\{E, F\} = \{E_1, E_d\}$ (in last case the eigenvalue associated with H is θ_{d-1} and $k\theta_{d-1} = \theta_1\theta_d$).

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