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Finite dimensional approximations to Wiener measure and path integral formulas on manifolds. (English) [Zbl 0943.58024](#)

J. Funct. Anal. 165, No. 2, 430-498 (1999).

The authors develop certain natural geometric approximation schemes for the Wiener measure on a compact Riemannian manifold. These approximations mimic the informal path integral formulas used for representing the heat semigroup on Riemannian manifolds. The approximation of the Wiener measure is made by measures on spaces of piecewise geodesics [see also *M. A. Pinsky*, Probabilistic analysis and related topics, Vol. 1, 199-236 (1978; [Zbl 0452.60083](#)) or *R. W. R. Darling*, Stochastics 12, 277-301 (1984; [Zbl 0543.58028](#))]. The authors interpret $\mathcal{D}\sigma$ in the heuristic expression of the Wiener measure $d\nu(\sigma) = \frac{1}{Z} e^{-(1/2)E(\sigma)} \mathcal{D}(\sigma)$, as a Riemannian volume form relative to a suitable metric. The authors use the Wong-Zakai type approximation theorem [*E. Wong* and *M. Zakai*, Int. J. Eng. Sci. 3, 213-229 (1965; [Zbl 0131.16401](#))] for stochastic differential equations instead of weak convergence arguments. This fact allows them to get a stronger form of convergence which is needed in the proof of the integration by parts formula for the Wiener measure.

Reviewer: [Mihai Gradinaru \(Nancy\)](#)

MSC:

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