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Nontrivial solutions for some fourth order semilinear elliptic problems. (English)

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From the introduction: Let us consider the following problem:

$$\begin{cases} \Delta^2 u + a^2 \Delta u = b[(u+1)^+ - 1] & \text{in } \Omega, \\ \Delta u = 0, \quad u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Δ^2 is the biharmonic operator, $u^+ = \max\{u, 0\}$, $\Omega \subset \mathbb{R}^N$ is a smooth open bounded set and a, b are constants.

We study the problem, when the nonlinearity $(u+1)^+ - 1$ is replaced by a more general function g, by using a variational approach. If $\lambda_1 < a^2$ and $b < \lambda_1\ (\lambda_1 - a^2)$ the existence of two solutions is proved by the classical mountain pass theorem. In two different cases we get the existence of two solutions using a "variation of linking" theorem, by studying the geometry of the functional. The first case is when $\lambda_{j+1} \leq a^2$ and b is close to $\lambda_{j+1}\ (\lambda_{j+1} - a^2)$ with $b < \lambda_{j+1}\ (\lambda_{j+1} - a^2)$ for some $j \geq 1$. The second case is when $\lambda_1 \leq a^2 \leq \lambda_i$ and b is close to $\lambda_i\ (\lambda_i - a^2)$ with $b > \lambda_i\ (\lambda_i - a^2)$ for some $i \geq 2$. Finally, we give some uniqueness results.

MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
35A15 Variational methods applied to PDEs
35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs

Keywords:

multiple solutions; uniqueness result; mountain pass theorem; variation of linking; travelling waves

Full Text: DOI

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