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Differential Galois theory and non-integrability of Hamiltonian systems. (English)

Zbl 0934.12003
Progress in Mathematics (Boston, Mass.) 179. Basel: Birkhäuser (ISBN 3-7643-6078-X/hbk; 978-3-0348-9741-9/pbk). xiv, 165 p. (1999).

A *Hamiltonian system* X_H on a symplectic manifold M (with symplectic structure coming from the closed non-degenerate two-form Ω) corresponding to the function H on M is the vector field such that $\Omega(X_H, \cdot) = dH$. Suppose the dimension of M is $2n$. A Hamiltonian system X_H on M is *completely integrable* if there are n functions $f_1 = H, f_2, \ldots, f_n$ on M such that

– There is a dense open subset of M over which the one forms df_i, $1 \leq i \leq n$ are linearly independent.

– All the Poisson brackets $\{f_i, f_j\} = \Omega(X_f, X_g)$ are 0.

Suppose further that M is a complex analytic manifold and that X_H is holomorphic. Given the (germ of) an integral curve of X_H, one can consider the corresponding complex curve in M as an abstract Riemann surface Γ along with an immersion $\Gamma \to M$, and the pullback of X_H to Γ then defines a linear differential operator L over the (differential) function field of Γ. If X_H is completely integrable, the work of the author and his collaborators shows that the differential Galois group G of L has a solvable connected component G^0. Conversely, if G^0 is not solvable, then X_H is not completely integrable. This important non-integrability criterion has been applied in a number of important applications, including the Bianchi IX cosmological model and Sitnikov's Three-Body Problem.

In the volume under review, the author presents this theory along with background information and examples. After a brief introductory chapter, he gives an account of differential Galois theory, a discussion of Hamiltonian systems, presents the non-integrability theorems, and then presents the applications. A short concluding chapter gives complementary results and conjectures for future research.

The volume is reasonably self contained, although readers meeting either differential Galois theory or Hamiltonian dynamical systems here for the first time will find the going rather difficult. The author includes more elementary references suitable for self-study in his 115 item bibliography, and provides directions to them at appropriate points in the book. For readers already prepared in the two prerequisite subjects, the author has provided a logically accessible account of a remarkable interaction between differential algebra and dynamics.

Reviewer: A.R.Magid (Norman)

MSC:

12H05	Differential algebra
37J30	Obstructions to integrability for finite-dimensional Hamiltonian and Lagrangian systems (nonintegrability criteria)
34M15	Algebraic aspects (differential-algebraic, hypertranscendence, group-theoretical) of ordinary differential equations in the complex domain
34-02	Research exposition (monographs, survey articles) pertaining to ordinary differential equations
12-02	Research exposition (monographs, survey articles) pertaining to field theory
34C45	Invariant manifolds for ordinary differential equations
37-02	Research exposition (monographs, survey articles) pertaining to dynamical systems and ergodic theory

Keywords:

differential Galois extension; Hamiltonian system; differential Galois group; non-integrability criterion; non-integrability theorems