Damelin, S. B.; **Kuijlaars, A. B. J.**

The support of the equilibrium measure in the presence of a monomial external field on $[-1,1]$. (English) Zbl 0943.31001

Trans. Am. Math. Soc. 351, No. 11, 4561-4584 (1999).

Corresponding to a continuous function $Q:[-1,1]\to\mathbb{R}$, there is a unique Borel probability measure μ on $[-1,1]$, called the equilibrium measure, such that, for some constant F, we have $U^\mu+Q\geq F$ on $[-1,1]$ with equality on $\operatorname{supp}\mu$; here $U^\mu(x)=-\int\log|x-t|d\mu(t)$. The authors study the support S of μ in the case where $Q(x)=-cx^{2m+1}$ with $m\in\mathbb{N}$ and $c>0$. Their main result asserts that corresponding to each m there are three critical values c_j with $0<c_1<c_2<c_3$ such that (i) for $0<c\leq c_1$, we have $S=[-1,1]$, (ii) for $c_1<c\leq c_2$, we have $S=[a,1]$, where $-1<a<0$, (iii) for $c_2<c<c_3$, we have $S=[a_1,b_1]\cup[a_2,1]$, where $-1<a_1<b_1<a_2<1$, (iv) for $c_3\leq c$, we have $S=[a,1]$, where $0<a<1$. The numbers a, a_1, a_2, b_1 depend on c. This result answers a question of *P. Deift, T. Kriecherbauer* and *K. T.-R. McLaughlin* [J. Approx. Theory 95, 388-475 (1998; Zbl 0918.31001)].

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MSC:

31A15 Potentials and capacity, harmonic measure, extremal length and related notions in two dimensions
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis

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