Berthier, M.; **Meziani, R.**; **Sad, P.**

On the classification of nilpotent singularities. (English) Zbl 0963.32021

Bull. Sci. Math. 123, No. 5, 351-370 (1999).

This paper deals with the analytic classification of germs of singular holomorphic foliations in a neighborhood of the origin of $\mathbb{C}^2$ having nonzero linear part in the "nilpotent" case.

Several works have already dealt with the "semisimple" case: *A. D. Bryuno* [Trans. Moscow Math. Soc. 25, 131-288 (1973); translation from Tr. Mosk. Mat. O.-va 25, 119-262 (1971; Zbl 0263.34003)]; *J. Ecalle* [Publ. Math. Orsay 85-05, 1-585 (1985; Zbl 0602.30029]; *J. Martinet* and *J.-P. Ramis* [Inst. Hautes Etud. Sci., Publ. Math. 55, 63-164 (1982; Zbl 0546.58038); Ann. Sci. Éc. Norm. Supér., IV. Ser. 16, 571-621 (1983; Zbl 0534.34011)]; *R. Perez-Marco* and *J.-C. Yoccoz* [Astérisque 222, 345-371 (1994; Zbl 0809.32008)].

In the "semisimple" case, the ratio of the eigenvalues of the linear part is closely related to the classification, and the moduli are essentially the ones of a holonomy diffeomorphism. Now the authors suppose that both eigenvalues vanish. They first consider a "saddle-node singularity" with two separatrices, where the strong separatrix is the x-axis and the central one is the y-axis. Then they give a new proof of the result that the holonomy diffeomorphism associated to the strong separatrix gives the classification *J. Martinet* and *J.P. Ramis* [loc. cit.]. The germ of a foliation is called nilpotent by the authors if its 1-jet is linearly equivalent to ydy, which possesses normal formal form of the type $\Omega^{n,p} = d(y^2 + x^n) + x^p U(x)dy$, where $n-1 \geq 2$ is the Milnor number, $p \geq 2$ is an integer and U is a an element of $\mathbb{C}[[x]]$ with $U(0) \neq 0$. For this kind of germs, if $2p < n$ and a desingularization process produces a saddle-node singularity, then they can characterize the holomorphic equivalence and the rigidity (rigidity holds precisely when holonomy group is non-abelian).

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32S05 Local complex singularities

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