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An application of the Parrott's theorem to the geometry of the unit sphere. (English)

Zbl 0936.47006
J. Math. Anal. Appl. 237, No. 2, 698-709 (1999).

Let $\mathcal{H},\mathcal{K}$ be Hilbert spaces and T be a 2×2 operator matrix acting on $\mathcal{H}\oplus\mathcal{K}$, with three entries, $A\in\mathcal{B}(\mathcal{H})$, $B\in\mathcal{B}(\mathcal{K},\mathcal{H})$, $C\in\mathcal{B}(\mathcal{H},\mathcal{K})$, specified and one unknown entry. *S. Parrott*'s theorem [J. Func. Anal. 30, 311-325 (1978; Zbl 0409.47004)] asserts that T has a contraction extension iff $AA^*+BB^*\le 1$ and $AA^*+CC^*\le 1$. Let $\mathcal{H}_i$ be Hilbert spaces, $\widetilde{H}=\oplus_{i=1}^n\mathcal{H}_i$, and $T=(T_{jk}),S=(S_{jk})$, $1\le j,k\le n$, be operator matrices acting on $\widetilde{\mathcal{H}}$. If $Z=(z_{jk})$, $1\le j,k\le n$, is a scalar matrix let $Z*S=(z_{jk}S_{jk})$, $1\le j,k\le n$. An operator matrix $T=(T_{jk})$ with $\|T\|=1$, is called a matrix extreme point of the unit sphere of $\mathcal{B}(\widetilde{\mathcal{H}})$ if $\|T+Z*S\|\le 1$, for any scalar matrix $Z=(z_{jk})$ with $|z_{jk}|\le 1$, implies $S=0$. This notion extends that of complex extreme point [see *V. I. Istrăţescu*, "Strict convexity and complex strict convexity", M. Dekker, New York (1984; Zbl 0538.46012)].

The author gives several characterizations of matrix extreme points whose proofs are based on Parrott's theorem mentioned above. As application he gives a matrix operator version of a theorem of *E. Thorp* and *R. Whitley* [Proc. Am. Math. Soc. 18, 640-646 (1967; Zbl 0185.20102)], on strong maximum modulus theorem for analytic functions into Banach spaces.

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MSC:

47A56 Functions whose values are linear operators (operator- and matrix-valued functions, etc., including analytic and meromorphic ones)
47A20 Dilations, extensions, compressions of linear operators

Keywords:

operator matrices; matrix extreme points; operator matrix valued analytic functions; contraction extension; maximum modulus theorem

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