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System equivalence for AR-systems over rings – with an application to delay-differential systems. (English) Zbl 0951.93015

Math. Control Signals Syst. 12, No. 3, 219-244 (1999).

Auto-regressive (AR) systems with defining equations described by matrices over an arbitrary integral domain $\mathcal{R}$ are studied in this paper. The results form a generalization of AR-systems over $\mathbb{R}[s]$ or $\mathbb{R}[s,s^{-1}]$ in a way reminiscent of the extension of the theory of state space systems over fields to the ring case.

In the behavioral approach to dynamical systems, a system is described as a triple $(T,W,\mathcal{B})$, where T is the time-axis, W is the space in which the signals take their values and $\mathcal{B}$ (the behavior) is a subspace of the signal space W^T. The behavior is then considered as the set of all time-trajectories which satisfy the governing laws of the system. The set-up in this paper is as follows: Let $\mathcal{R}$ be an integral domain and let $\mathcal{M}$ be a module over $\mathcal{R}$, consisting of all (one-variable) trajectories under consideration. Each ring element is associated with an operator acting on the elements of the module $\mathcal{M}$. This set-up differs from the behavioral approach in that a time-axis T and a space W are not explicitly described. Instead, $\mathcal{M}^q$ is taken as the signal space, allowing endowment with a richer structure.

The goal of the paper is to characterize all matrices over $\mathcal{R}$ describing the same behavior for a given module $\mathcal{M}$ over $\mathcal{R}$. This problem is tackled by defining a ring extension $\mathcal{R}_M$ of $\mathcal{R}$ which explicitly depends on $\mathcal{M}$. It turns out that $\mathcal{M}$ is also a module over $\mathcal{R}_\mathcal{M}$ and that a system $(P,\mathcal{M}^q)$ over $\mathcal{R}$, where $P\in\mathcal{R}^{p\times q}$, can also be considered as a system over $\mathcal{R}_\mathcal{M}$. This allows greater freedom of manipulation, provided $\mathcal{R}$ is indeed properly contained in $\mathcal{R}_\mathcal{M}$. A sufficient condition for equality of these is found.

For the study of system equivalence, the module $\mathcal{M}$ is assumed to be divisible (a rather restrictive assumption, not satisfied by several discrete-time systems). Necessary and sufficient conditions for equivalence are found, first in the case of square matrices and then also in the general case.

The paper concludes with an application to differential systems, where the ring $\mathcal{R}_\mathcal{M}$ is characterized explicitly.

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MSC:

93B25 Algebraic methods

34K35 Control problems for functional-differential equations

Keywords:

autoregressive systems (AR-systems); systems over rings; delay-differential systems; behavioral approach; system equivalence

Full Text: DOI