

Tomiyama, Masato

Weakly Q -polynomial distance-regular graphs. (English) Zbl 0935.05101
RIMS Kokyuroku 1063, 104-110 (1998).

This note is based on the author's paper [A note on the primitive idempotents of distance-regular graphs (preprint)]. In the first section, we state our results. In the next section, we review some definitions and basic concepts. For more background information, the reader may refer to *E. Bannai* and *T. Itô* [Algebraic combinatorics. I: Association schemes (1984; Zbl 0555.05019)], *A. E. Brouwer*, *A. M. Cohen* and *A. Neumaier* [Distance-regular graphs (1989; Zbl 0747.05073)] or *C. D. Godsil* [Algebraic combinatorics (1993; Zbl 0784.05001)].

Let Γ denote a distance-regular graph with diameter $d \geq 3$ and eigenvalues $\theta_0 > \theta_1 > \dots > \theta_d$. Let E and F denote nontrivial primitive idempotents of Γ . In [Tight graphs and their primitive idempotents, J. Algebr. Comb. 10, No. 1, 47-59 (1999; Zbl 0927.05085)], *A. A. Pascasio* investigated the situation that $E \circ F$ is a scalar multiple of a primitive idempotent H of Γ . She showed this occurs exactly when Γ is either bipartite or tight (in the sense of *A. Jurišić*, *J. Koolen* and *P. M. Terwilliger* [Tight distance-regular graphs (preprint)]). Moreover, she showed that at least one of E and F is equal to E_d if Γ is bipartite, and that E and F are a permutation of E_1 and E_d if Γ is tight. If Γ is bipartite, *M. Lang* obtained an inequality involving the cosines of E , and showed that equality is closely related to Γ being Q -polynomial with respect to E . See [*M. Lang*, A new inequality for bipartite distance-regular graphs (preprint)]. If Γ is tight, *A. A. Pascasio* obtained similar inequalities involving the cosines of E , and showed that again equality is closely related to Γ being Q -polynomial with respect to E . See [*A. A. Pascasio*, Tight distance-regular graphs and the Q -polynomial property (preprint)].

In this note, we investigate a slightly more general situation. Let E denote a nontrivial primitive idempotent of Γ and let $F \in \{E_1, E_d\}$. Our situation is that there exists a primitive idempotent H of Γ such that $E \circ F$ is a linear combination of F and H . Our main purpose is to obtain the above inequalities under our general assumption, and to show that again equality is closely related to Γ being Q -polynomial with respect to E .

MSC:

05E30 Association schemes, strongly regular graphs

Keywords:

distance-regular graph; eigenvalues; Q -polynomial