

[Bressoud, David M.](#)

Proofs and confirmations. The story of the alternating sign matrix conjecture. (English)

Zbl 0944.05001

Spectrum Series. Cambridge: Cambridge University Press. xv, 274 p. (1999).

The main characters of this story are the alternating sum matrices. An alternating sign matrix is a square matrix of 0s, 1s and -1 s for which the sum of the entries in each row and column is 1 and the non-zero entries in each row and column alternate in sign. The book includes the interesting bit of mathematical history culminating in the alternating sign conjecture: The total number of $n \times n$ alternating sign matrices is

$$\prod_{j=0}^{n-1} \frac{(3j+1)!}{(n+j)!}.$$

As the author explains: “This story is woven on the strands of fourteen conjectures that I have chosen from the many that arose in the course of investigations of alternating sign matrices. All but two of these have been proven. Many other related conjectures are still open problems. We shall see the details of many of the proofs, but they are less important than the themes that they draw upon. This is the power of a good conjecture and the reason for seeking its proof: It can reveal unexpected connections and guide us to the ideas that are truly important.”

The book has seven chapters. The basic definitions and the fourteen conjectures are introduced in the first chapter. The combinatorial structures needed to settle these conjectures are then developed in the next four chapters entitled Fundamental Structures, Lattice Paths and Plane Partitions, Symmetric Functions and Hypergeometric Series. The alternating sign conjecture is proved in Chapter 6, Explorations and the “refined” alternating sign conjecture is proved in the last chapter, Square Ice.

Reviewer: [J.E.Graver \(Syracuse\)](#)

MSC:

- [05-01](#) Introductory exposition (textbooks, tutorial papers, etc.) pertaining to combinatorics
- [05A17](#) Combinatorial aspects of partitions of integers
- [05E05](#) Symmetric functions and generalizations
- [82B20](#) Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
- [11P81](#) Elementary theory of partitions

Cited in **169** Documents

Keywords:

[alternating sum matrices](#)