Savin, A. Yu.; **Sternin, B. Yu.**
Elliptic operators in even subspaces. (English. Russian original) ⟦Zbl 0963.58008⟧
Sb. Math. 190, No. 8, 1195-1228 (1999); translation from Mat. Sb. 190, No. 8, 125-160 (1999).

The authors study the index problem when an elliptic operator is restricted to certain subspaces of smooth sections of vector bundles over closed manifolds, as for the index problem on manifolds with boundary, where boundary conditions reduce the space of sections to be considered. The subspaces considered in this paper, which are called even subspaces, are the image of zero order pseudodifferential projections whose principal symbols are even functions on the cotangent bundle.

Let P_1, P_2 be two such projections, and let D be a pseudodifferential operator such that the restriction $D : \mathrm{Im}P_1 \to \mathrm{Im}P_2$ is defined. The authors define the "principal symbol" of such restriction, and such restriction is called elliptic when its principal symbol is an isomorphism at every point of M. They not only show that ellipticity is a criterion for the Fredholm property, but also prove an index formula for this kind of operators. Namely, they show that its analytical index equals the sum of two homotopy invariant terms. The first term is defined only by the principal symbol, while the second term is defined only by the given even subspaces. Precisely, this second term can be regarded as a relative index of P_1, P_2; it can be written as $d([P_1] - [P_2])$ for a real valued "dimension" homomorphism d on the Grothendieck group of the semigroup of "equivalent" even projections. It is shown that the index theorem for a general boundary value problem fits into this theory. Moreover d is closely connected with the theory of η-invariants of selfadjoint elliptic operators of even order on odd-dimensional manifolds.

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MSC:

58J05	Elliptic equations on manifolds, general theory
58J28	Eta-invariants, Chern-Simons invariants
35S10	Initial value problems for PDEs with pseudodifferential operators
47A53	(Semi-) Fredholm operators; index theories
55R50	Stable classes of vector space bundles in algebraic topology and relations to K-theory

Cited in **3** Reviews
Cited in **6** Documents

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