Crawley-Boevey, William

Projective algebras, differential operators and a Conze embedding for deformations of Kleinian singularities. (English) Zbl 0958.16014

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In the paper under review the author continues the study of deformations of Kleinian singularities begun in an earlier joint paper with M. P. Holland. The main tool used in that paper was a new class of algebras called deformed preprojective algebras. In the present paper, the author provides a conceptual approach to the study of deformed preprojective algebras by constructing a wider class of algebras.

This is done as follows. Let A be an associative algebra over a field k and A^e the enveloping algebra of A. Let $\Delta \in {}^*\mathrm{Der}(A, A^e)$ be the derivation defined by $\Delta(a) := a \otimes 1 - 1 \otimes a$. The space ${}^*\mathrm{Der}(A, A^e)$ is naturally an A^e-module, and therefore the tensor algebra $T_A{}^*\mathrm{Der}(A, A^e)$ can be formed. Now, for any $a \in A$, define $\Pi^a(A) := T_A{}^*\mathrm{Der}(A, A^e)/(\Delta - a)$. It turns out, that the algebra $\Pi^a(A)$ depends only on the class of a in $H_0(A) = A/[A, A]$. The latter comes equipped with the trace map ${}^*\mathrm{tr}\colon K_0(A) \to A/[A, A]$, which sends a projective P to the trace of any idempotent in $M_n(A)$ whose image is isomorphic to P. This map extends to a linear map $k \otimes_{\mathbb{Z}} K_0(A) \to A/[A, A]$, also denoted ${}^*\mathrm{tr}$. Now, if $\lambda \in k \otimes_{\mathbb{Z}} K_0(A)$, define

$$\Pi^\lambda(A) := \Pi^{a_\lambda}(A) = T_A{}^*\mathrm{Der}(A, A^e)/(\Delta - a_\lambda),$$

where a_λ is any lift of the trace of λ to A. The algebra $\Pi^\lambda(A)$ exhibits a number of remarkable properties.

1. If A is finite-dimensional and hereditary, then $\Pi^0(A)$ is a preprojective algebra, as defined by Baer, Geigle, and Lenzing.

2. If A is the path algebra kQ of a quiver Q and $\lambda \in k \otimes_{\mathbb{Z}} K_0(kQ)$, then $\Pi^\lambda(A)$ is a deformed preprojective algebra, as defined by Crawley-Boevey and Holland. In particular, if Q is the quiver with one vertex and one loop and if $\nu \in k$, then $\Pi^\nu(k[X]) \cong k\langle X, Y \mid XY - YX = \nu\rangle =: C_\nu$.

3. If A is the coordinate ring of a smooth affine curve over k, then $\Pi^0(A)$ is the coordinate ring of the cotangent bundle of Spec A and $\Pi^1(A)$ is the ring of differential operators for A.

After a number of other properties of $\Pi^\lambda(A)$ are established, the author applies the developed technique to the study of deformations of the path algebras of the extended Dynkin quivers Q over an algebraically closed k. Under these assumptions, $\Pi^\lambda(kQ)$ is a prime Noetherian ring of Gelfand-Kirillov dimension 2 and there exists an injective map $\theta_\lambda\colon \Pi^\lambda(kQ) \to M_N(C_\nu)$. Here v is the linear combination of the components of the minimal positive imaginary root for Q with coefficients being the components of λ and C_ν is the algebra defined in the second example above. As an application, a family of deformations $\mathcal{O}^\lambda$ of a Kleinian singularity, introduced and studied in an earlier paper of Crawley-Boevey and Holland, is embedded in C_ν. This embedding induces an isomorphism on quotient division rings.

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MSC:

16G20	Representations of quivers and partially ordered sets
16S32	Rings of differential operators (associative algebraic aspects)
16S80	Deformations of associative rings
14B07	Deformations of singularities

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