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Lagnese, J. E.; **Leugering, G.**

Dynamic domain decomposition in approximate and exact boundary control in problems of transmission for wave equations. (English) Zbl 0952.93010

SIAM J. Control Optimization 38, No. 2, 503-537 (2000).

This is a paper containing several new results and extending much of what has been developed in the last decade. The primary theme is dynamic domain decomposition for optimal boundary control. A final value control problem is defined for wave propagation in a heterogeneous medium. In a medium for which the material properties are almost constant in each separate region of the domain this problem is closely related to the transmission problem. Then the decomposition problem consists of handling separately each subdomain in which the material properties are constant, and communication between these separate subdomains is restored to match material properties by means of iteration. In the limit control laws are restored globally.

Two levels of difficulty have to be reconciled. Controllability (and observability) properties are relatively easy to determine at each local subdomain level. On the other hand complex structure containing shells, beams, strings, etc. is difficult to handle at the subdomain level. It is natural to discretize such complex systems and treat them in a purely numerical manner. The procedure established by Glowinski, Lions and others consists of running such different iterative processes in parallel. For details of such technique see for example the paper of *A. Bamberger, R. Glowinski* and *Q. H. Tran* [SIAM J. Numer. Anal. 34, 603-639 (1997; Zbl 0877.35066)], in which the authors compute acoustic wave propagation in a medium with discontinuous properties across boundaries of subdomains. Let Ω_i be the ith subdomain, $\Omega = \cup\Omega_i$, $\Gamma = \partial\Omega$, Γ_{ij} is the boundary between ith and jth subdomain. Here the authors consider a system obeying the transmission laws:

For each ith subdomain Ω_i we have $y_{i,tt} - a_i\Delta y_i = 0$ in $\Omega_i \times [0,T]$, $a_m\partial y_m/\partial\nu_m = f \in L^2$ on $\Gamma \times [0,T]$, and on Γ_{ij} we have matching properties: $y_i = y_j$, $a_i\partial y_i/\partial\nu_i = -a_j\partial y_j/\partial\nu_j$ on Γ_{ij}, $y_i = \partial y_i/\partial t = 0$ at $t = 0$. Now the problem consists in finding the optimality criteria for the control f, minimizing a given cost function, which measures deviation of the final state from a desired configuration. A criterion is given in terms of the adjoint state, following standard control theory practice.

At this point the real work of decomposition of the domain begins. Local optimal control is established for each subdomain for a fixed value of the penalty parameter. Both the approximate controllability and exact controllability are considered. The local optimality and global optimality are derived by suitable domain decomposition and saddle point iteration. The penalty parameter increases without bound as solutions of local optimality problems are driven towards a global optimality.

This brief review misses many important arguments such as for example comparisons of the technique used here with the alternate approach of using Hilbert's uniqueness method (HUM), the use of Schwartz alternating iteration method, improvements on some domain decomposition results of J.-D. Benamou.

Reviewer: Vadim Komkov (Florida)

MSC:

93B05	Controllability
93B40	Computational methods in systems theory (MSC2010)
49N10	Linear-quadratic optimal control problems
65K10	Numerical optimization and variational techniques
35L05	Wave equation

Keywords:

domain decomposition; optimal control; controllability; saddle-point iteration; acoustic wave propagation; local optimality; global optimality

Full Text: DOI