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A multilevel characteristics method for periodic convection-dominated diffusion problems.

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The authors consider the linear unsteady convection-diffusion problem that involves a function $u(x, t)$ from $]0, 2\pi[^d \times \mathbb{R}^+$ into \mathbb{R} :

$$\frac{\partial u}{\partial t} + a \cdot \nabla u - \nu \Delta u = f,$$

where ν is the viscosity, f is the forcing term and a is some divergence-free vector field. The initial conditions are that $u(x, 0) = u_0(x)$ for all $x \in]0, 2\pi[^d$ and u is 2π -periodic in all space variables.

The spatial discretization uses the space S_M of all trigonometric polynomials of degree $\leq M$ in each variable. A two-level decomposition is introduced by considering another parameter m , $0 < m < M$ and writing $S_M = S_m + (I - P_m)S_M$ where P_m is the L^2 -projection onto S_m .

The numerical integration then relies on two different interpolation operators and the approximate solution is obtained as the sum of a term in S_m and a term in $(I - P_m)S_M$. The two components are advanced in time using different time steps.

The authors investigate the stability of this scheme and derive error estimates. These indicate that the high-frequency term can be integrated with a larger time-step.

The paper contains a few numerical tests for 1D-problems. It is found that the two-level method allows a significant gain in computing time with respect to the classical method. The accuracy is always better than the one of the classical method based on S_m and close to the one based on S_M .

Reviewer: [Willy Govaerts \(Gent\)](#)

MSC:

[65M25](#) Numerical aspects of the method of characteristics for initial value and initial-boundary value problems involving PDEs

Cited in 1 Document

[65M55](#) Multigrid methods; domain decomposition for initial value and initial-boundary value problems involving PDEs

[65M12](#) Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

[35K15](#) Initial value problems for second-order parabolic equations

[65M15](#) Error bounds for initial value and initial-boundary value problems involving PDEs

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[multilevel method](#); [spectral method](#); [numerical examples](#); [method of characteristics](#); [periodic solution](#); [convection-diffusion problem](#); [stability](#); [error estimates](#)

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