Trudinger, Neil S.; **Wang, Xu-Jia**

Hessian measures. II. (English) Zbl 0947.35055

Ann. Math. (2) 150, No. 2, 579-604 (1999).

In a previous paper [Topol. Methods Nonlinear Anal. 10, No. 2, 225-239 (1997; Zbl 0915.35039)] the same authors introduced the notion of k-Hessian measures associated with a continuous k-convex function in a domain $\Omega \subset \mathbb{R}^n$, $k = 1, \dots, n$, and proved a weak continuity result with respect to local uniform convergence. In the present paper they consider upper semicontinuous k-convex functions and prove weak continuity of the corresponding k-Hessian measure with respect to convergence in measure. To get this result, they first prove some lemmas and theorems for k-convex functions which may have own interest. Then, some local integral estimates for the k-Hessian operator $F_k[u]$ and for the gradient Du in terms of the integral of $|u|$ are proved. Using the above results, the following interesting theorem is proved: For any k-convex function u, there exists a Borel measure $\mu_k[u]$ in Ω such that: (i) $\mu_k[u] = F_k[u]$ for $u \in C^2(\Omega)$, and (ii) if $\{u_m\}$ is a sequence of k-convex functions converging locally in measure to a k-convex function u, the sequence of Borel measures $\{\mu_k[u]\}$ converges weakly to $\mu_k[u]$.

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MSC:

35J60 Nonlinear elliptic equations
28A33 Spaces of measures, convergence of measures
35B05 Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs
31B15 Potentials and capacities, extremal length and related notions in higher dimensions

Keywords:

k-Hessian measures; k-convex functions

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