

Vanden Eynde, Ria

Development of the concept of homotopy. (English) [Zbl 0944.55001](#)

James, I. M. (ed.), History of topology. Amsterdam: Elsevier. 65-102 (1999).

Introduction: Homotopy is concerned with the identification of geometric objects (at first, paths) which can be continuously deformed into each other, these are then considered equivalent. The formal expression of this type of intuitive equivalence concept in terms of reflexivity, symmetry and transitivity was given in the late 1920's [*O. Schreier*, Abh. Math. Semin. Univ. Hamburg 5, 233-244 (1927; [JFM 53.0110.02](#)), pp. 233, 234] after a century-long history. The origins of the homotopy concept for paths can be found within analysis where it was used as a visual tool to decide whether two paths with the same endpoints would lead to the same result for integration, or analytic continuation of a multi-valued function. The mathematization of the intuitive equivalence concept thus depended upon the objectives of the mathematicians who used it. We will see how this caused an ambiguity around the homotopy concept practically from the moment it originated: it got confused with other kinds of equivalences which were not immediately recognized as being different. Gradually, certain descriptions of the homotopy concept came to the front: constrained deformation (in which one or both endpoints are fixed) became favoured compared to free deformation. Although the latter is a more intuitive concept, the former will prove to be more interesting: it will allow for the introduction of a group structure. This paper describes the history of the concept of homotopy of paths and its after-effects in, e.g., higher homotopy groups.

For the entire collection see [[Zbl 0922.54003](#)].

MSC:

- [55-03](#) History of algebraic topology
- [01A60](#) History of mathematics in the 20th century
- [55P99](#) Homotopy theory
- [55Q99](#) Homotopy groups
- [57T99](#) Homology and homotopy of topological groups and related structures

Cited in **2** Documents

Keywords:

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