

**Frenkel, Edward; Reshetikhin, Nicolai**

**The  $q$ -characters of representations of quantum affine algebras and deformations of  $W$ -algebras.** (English) [Zbl 0973.17015](#)

Jing, Naihuan (ed.) et al., Recent developments in quantum affine algebras and related topics. Proceedings of the international conference on representations of affine and quantum affine algebras and their applications, North Carolina State University, Raleigh, NC, USA, May 21-24, 1998. Providence, RI: American Mathematical Society (AMS). Contemp. Math. 248, 163-205 (1999).

Let  $\mathfrak{g}$  be a simple Lie algebra,  $\widehat{\mathfrak{g}}$  the corresponding non-twisted affine Kac-Moody algebra and  $U_q\widehat{\mathfrak{g}}$  the corresponding quantized universal enveloping algebra. Let  $\text{Rep}U_q\widehat{\mathfrak{g}}$  be the Grothendieck ring of the category of finite-dimensional representations of  $U_q\widehat{\mathfrak{g}}$ . By using the universal R-matrix of  $U_q\widehat{\mathfrak{g}}$  and an analogue of the Harish-Chandra homomorphism, the authors construct an injective homomorphism  $\chi_q : \text{Rep}U_q\widehat{\mathfrak{g}} \rightarrow \mathcal{Y} = \mathbb{Z}[Y_{i,a_i}^{\pm 1}]_{i=1,\dots,\ell; a_i \in \mathbb{C}^\times}$ , where  $Y_{i,a_i}^{\pm 1}$  are built from the Heisenberg algebra elements in the Drinfeld realization of  $U_q\widehat{\mathfrak{g}}$ . The authors show that  $\chi_q$  behaves well with respect to the restriction to  $U_q\mathfrak{g}$  and to the quantum affine subalgebras of  $U_q\widehat{\mathfrak{g}}$ . The authors call  $\chi_q(V)$  the  $q$ -character of representation  $V$ . This construction is motivated by the theory of deformed  $\mathcal{W}$ -algebras [*E. Frenkel and N. Reshetikhin*, Commun. Math. Phys. 178, 237-264 (1996; [Zbl 0869.17014](#))], and the authors conjecture that the image of  $\chi_q$  equals the intersection of certain screening operators  $S_i$  defined on  $\mathcal{Y}$ . The  $q$ -characters are closely related to a character theory in [*H. Knight*, J. Algebra 174, 187-196 (1995; [Zbl 0868.17009](#))].

For the entire collection see [[Zbl 0932.00043](#)].

Reviewer: [M.Princ \(Zagreb\)](#)

**MSC:**

- [17B37](#) Quantum groups (quantized enveloping algebras) and related deformations
- [81R50](#) Quantum groups and related algebraic methods applied to problems in quantum theory
- [17B68](#) Virasoro and related algebras
- [81R10](#) Infinite-dimensional groups and algebras motivated by physics, including Virasoro, Kac-Moody,  $W$ -algebras and other current algebras and their representations

Cited in **12** Reviews  
Cited in **105** Documents

**Keywords:**

non-twisted affine Kac-Moody algebra; quantized universal enveloping algebra; Harish-Chandra homomorphism; injective homomorphism; screening operators;  $q$ -characters

**Full Text:** [arXiv](#)