Crawley-Boevey, William; Holland, Martin P.
Noncommutative deformations of Kleinian singularities. (English) Zbl 0974.16007
Duke Math. J. 92, No. 3, 605-635 (1998).

A Kleinian singularity is the quotient K^2/Γ, where K is an (algebraically closed) field (of characteristic zero) and Γ is a nontrivial finite subgroup of $\mathrm{SL}_2(K)$. More precisely, this is an object whose coordinate ring is $K[x,y]^\Gamma$, where the action of Γ on $K[x,y]$ is extended from the given action of Γ on the two-dimensional vector space spanned by x and y. In the paper under review, the authors define and study a family $\mathcal{O}^\lambda$ of deformations of $K[x,y]^\Gamma$, where $\lambda \in Z(K\Gamma)$. The definition of $\mathcal{O}^\lambda$ is as follows. Γ acts in an obvious way on the noncommuting polynomials $K\langle x,y\rangle$ and one forms the corresponding skew group ring $K\langle x,y\rangle\Gamma$. For $\lambda \in Z(K\Gamma)$, define $\mathcal{S}^\lambda$ as the quotient $K\langle x,y\rangle\Gamma/(xy-yx-\lambda)$. Let $e \in K\Gamma$ be the average of the group elements. Then $\mathcal{O}^\lambda$ is defined as $e\mathcal{S}^\lambda e$. These rings are Noetherian, finitely generated K-algebras, of Gelfand-Kirillov dimension 2. They are also Auslander-Gorenstein and Cohen-Macaulay. Other properties of $\mathcal{O}^\lambda$ are studied by means of the so called deformed preprojective algebras.

In a subsequent paper by the second author [Comment. Math. Helv. 74, No. 4, 548-574 (1999; Zbl 0958.16014)], deformed preprojective algebras are embedded in a wider class of algebras, which provides a more conceptual approach to the study of deformations of Kleinian singularities. The reader is referred to that paper for more details.

Reviewer: Alex Martsinkovsky (Boston)

MSC:

16G10 Representations of associative Artinian rings
14B07 Deformations of singularities
16S80 Deformations of associative rings
14A22 Noncommutative algebraic geometry
16S32 Rings of differential operators (associative algebraic aspects)

Keywords:

Kleinian singularities; deformed preprojective algebras; McKay graphs; deformations

Full Text: DOI

References:

[1] D. Baer, W. Geigle, and H. Lenzing, The preprojective algebra of a tame hereditary Artin algebra , Comm. Algebra 15 (1987), no. 1-2, 425-457. · Zbl 0612.16015 · doi:10.1080/00927878708823425

[2] V. V. Bavula, Generalized Weyl algebras and their representations , Algebra i Analiz 4 (1992), no. 1, 75-97. · Zbl 0807.16027

[3] I. N. Bernšteĭ n, I. M. Gelfand, and V. A. Ponomarev, Coxeter functors, and Gabriel's theorem , Uspehi Mat. Nauk 28 (1973), no. 2(170), 19-33. · Zbl 0269.08001

[4] J.-E. Björk, Filtered Noetherian rings , Noetherian rings and their applications (Oberwolfach, 1983) ed. L. W. Small, Math. Surveys Monogr., vol. 24, Amer. Math. Soc., Providence, RI, 1987, pp. 59-97. · Zbl 0648.16001

[5] K. A. Brown and M. Lorenz, Grothendieck groups of invariant rings: linear actions of finite groups , Math. Z. 221 (1996), no. 1, 113-137. · Zbl 0905.19001 · doi:10.1007/BF02622103 · eudml:174838

[6] S. C. Coutinho and M. P. Holland, \(K\)-theory of twisted differential operators , J. London Math. Soc. (2) 47 (1993), no. 2, 240-254. · Zbl 0811.16020 · doi:10.1112/jlms/s2-47.2.240

[7] J. H. Cozzens, Maximal orders and reflexive modules , Trans. Amer. Math. Soc. 219 (1976), 323-336. · Zbl 0334.16009 · doi:10.2307/1997597

[8] V. Dlab and C. M. Ringel, Indecomposable representations of graphs and algebras , Mem. Amer. Math. Soc. 6 (1976), no. 173, v+57. · Zbl 0332.16015

[9] V. Dlab and C. M. Ringel, The preprojective algebra of a modulated graph , Representation theory, II (Proc. Second Internat. Conf., Carleton Univ., Ottawa, Ont., 1979) eds. V. Dlab and P. Gabriel, Lecture Notes in Math., vol. 832, Springer, Berlin, 1980, pp. 216-231. · Zbl 0489.16024

[10] V. Dlab and C. M. Ringel, Eigenvalues of Coxeter transformations and the Gelfand-Kirillov dimension of the preprojective

algebras , Proc. Amer. Math. Soc. 83 (1981), no. 2, 228-232. JSTOR: · Zbl 0471.15005 · doi:10.2307/2043500 · links.jstor.org

[11] D. Eisenbud, Commutative algebra:With a view towards algebraic Geometry , Graduate Texts in Mathematics, vol. 150, Springer-Verlag, New York, 1995. · Zbl 0819.13001

[12] P. Gabriel, Auslander-Reiten sequences and representation-finite algebras , Representation theory, I (Proc. Workshop, Carleton Univ., Ottawa, Ont., 1979) eds. V. Dlab and P. Gabriel, Lecture Notes in Math., vol. 831, Springer, Berlin, 1980, pp. 1-71. · Zbl 0445.16023

[13] I. M. Gelfand and V. A. Ponomarev, Model algebras and representations of graphs , Funktsional. Anal. i Prilozhen. 13 (1979), no. 3, 1-12. · Zbl 0437.16020

[14] T. G. Goodwillie, Relative algebraic \(K\)-theory and cyclic homology , Ann. of Math. (2) 124 (1986), no. 2, 347-402. · Zbl 0627.18004 · doi:10.2307/1971283

[15] T. J. Hodges, Noncommutative deformations of type-\(A\) Kleinian singularities , J. Algebra 161 (1993), no. 2, 271-290. · Zbl 0807.16029 · doi:10.1006/jabr.1993.1219

[16] M. P. Holland, \(K\)-theory of endomorphism rings and of rings of invariants , J. Algebra 191 (1997), no. 2, 668-685. · Zbl 0878.16007 · doi:10.1006/jabr.1996.6931

[17] J. E. Humphreys, Reflection groups and Coxeter groups , Cambridge Studies in Advanced Mathematics, vol. 29, Cambridge University Press, Cambridge, 1990. · Zbl 0725.20028

[18] A. Joseph, A generalization of Quillen's lemma and its application to the Weyl algebras , Israel J. Math. 28 (1977), no. 3, 177-192. · Zbl 0366.17006 · doi:10.1007/BF02759808

[19] V. G. Kac, Infinite-dimensional Lie algebras,3rd ed. , Cambridge University Press, Cambridge, 1990. · Zbl 0716.17022

[20] H. Kraft, Geometrische Methoden in der Invariantentheorie , Aspects of Mathematics, D1, Friedr. Vieweg & Sohn, Braunschweig, 1984. · Zbl 0569.14003

[21] G. Krause and T. H. Lenagan, Growth of algebras and Gelfand-Kirillov dimension , Research Notes in Mathematics, vol. 116, Pitman (Advanced Publishing Program), Boston, MA, 1985. · Zbl 0564.16001

[22] P. B. Kronheimer, 1987, letter to P. Slodowy.

[23] P. B. Kronheimer, The construction of ALE spaces as hyper-Kähler quotients , J. Differential Geom. 29 (1989), no. 3, 665-683. · Zbl 0671.53045 ·

[24] L. Le Bruyn and C. Procesi, Semisimple representations of quivers , Trans. Amer. Math. Soc. 317 (1990), no. 2, 585-598. JSTOR: · Zbl 0693.16018 · doi:10.2307/2001477 · links.jstor.org

[25] G. Lusztig, Quivers, perverse sheaves, and quantized enveloping algebras , J. Amer. Math. Soc. 4 (1991), no. 2, 365-421. JSTOR: · Zbl 0738.17011 · doi:10.2307/2939279 · links.jstor.org

[26] R. Martin, Skew group rings and maximal orders , Glasgow Math. J. 37 (1995), no. 2, 249-263. · Zbl 0830.16018 · doi:10.1017/S0017089500031153

[27] J. C. McConnell and J. C. Robson, Noncommutative Noetherian rings , Pure and Applied Mathematics, John Wiley & Sons Ltd., Chichester, 1987. · Zbl 0644.16008

[28] J. McKay, Graphs, singularities, and finite groups , The Santa Cruz Conference on Finite Groups (Univ. California, Santa Cruz, Calif., 1979), Proc. Sympos. Pure Math., vol. 37, Amer. Math. Soc., Providence, R.I., 1980, pp. 183-186. · Zbl 0451.05026

[29] S. Montgomery and L. W. Small, Fixed rings of Noetherian rings , Bull. London Math. Soc. 13 (1981), no. 1, 33-38. · Zbl 0453.16021 · doi:10.1112/blms/13.1.33

[30] D. Mumford, J. Fogarty, and F. Kirwan, Geometric invariant theory , Ergebnisse der Mathematik und ihrer Grenzgebiete (2) [Results in Mathematics and Related Areas (2)], vol. 34, Springer-Verlag, Berlin, 1994. · Zbl 0797.14004 · doi:10.1007/978-3-642-57916-5

[31] H. Nakajima, Instantons on ALE spaces, quiver varieties, and Kac-Moody algebras , Duke Math. J. 76 (1994), no. 2, 365-416. · Zbl 0826.17026 · doi:10.1215/S0012-7094-94-07613-8 ·

[32] D. Quillen, Higher algebraic \(K\)-theory. I , Algebraic \(K\)-theory, I: Higher \(K\)-theories (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972), Springer, Berlin, 1973, 85-147. Lecture Notes in Math., Vol. 341. · Zbl 0292.18004

[33] I. Reiten and M. Van den Bergh, Two-dimensional tame and maximal orders of finite representation type , Mem. Amer. Math. Soc. 80 (1989), no. 408, viii+72. · Zbl 0677.16002

[34] P. Slodowy, Simple singularities and simple algebraic groups , Lecture Notes in Mathematics, vol. 815, Springer, Berlin, 1980. · Zbl 0441.14002

[35] L. Small and R. B. Warfield, Prime affine algebras of Gelfand-Kirillov dimension one , J. Algebra 91 (1984), no. 2, 386-389. · Zbl 0545.16011 · doi:10.1016/0021-8693(84)90110-8

[36] S. P. Smith, A class of algebras similar to the enveloping algebra of \(\mathrm sl(2)\) , Trans. Amer. Math. Soc. 322 (1990), no. 1, 285-314. · Zbl 0732.16019 · doi:10.2307/2001532

[37] M. Van den Bergh and F. Van Oystaeyen, Lifting maximal orders , Comm. Algebra 17 (1989), no. 2, 341-349. · Zbl 0663.16004 · doi:10.1080/00927878908823732

[38] C. A. Weibel, Module structures on the \(K\)-theory of graded rings , J. Algebra 105 (1987), no. 2, 465-483. · Zbl 0611.13012 · doi:10.1016/0021-8693(87)90210-9

[39] Z. Yi, Injective homogeneity and the Auslander-Gorenstein property , Glasgow Math. J. 37 (1995), no. 2, 191-204. · Zbl 0830.16010 · doi:10.1017/S0017089500031098

paper as accurately as possible without claiming the completeness or perfect precision of the matching.