

Silverman, Joseph H.

The space of rational maps on \mathbb{P}^1 . (English) Zbl 0966.14031

Duke Math. J. 94, No. 1, 41-77 (1998).

Introduction: The set of morphisms $\varphi : \mathbb{P}^1 \rightarrow \mathbb{P}^1$ of degree d is parametrized by an affine open subset Rat_d of \mathbb{P}^{2d+1} . In this paper, we consider the action of SL_2 on Rat_d induced by the conjugation action of SL_2 on rational maps; that is, $f \in \text{SL}_2$ acts on φ via $\varphi^f = f^{-1} \circ \varphi \circ f$. The quotient space $M_d = \text{Rat}_d / \text{SL}_2$ arises very naturally in the study of discrete dynamical systems on \mathbb{P}^1 . We prove that M_d exists as an affine integral scheme over \mathbb{Z} , that M_2 is isomorphic to $\mathbb{A}_{\mathbb{Z}}^2$ and that the natural completion of M_2 obtained using geometric invariant theory is isomorphic to $\mathbb{P}_{\mathbb{Z}}^2$. These results, which generalize results of Milnor over \mathbb{C} , should be useful for studying the arithmetic properties of dynamical systems.

MSC:

14L24 Geometric invariant theory

37K20 Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with algebraic geometry, complex analysis, and special functions

14L30 Group actions on varieties or schemes (quotients)

14E05 Rational and birational maps

Cited in **5** Reviews
Cited in **35** Documents

Keywords:

space of rational maps; action of special linear group; discrete dynamical system; geometric invariant theory

Full Text: [DOI](#)

References:

- [1] A. Altman and S. Kleiman, Introduction to Grothendieck Duality Theory , Lecture Notes in Math., vol. 146, Springer-Verlag, Berlin, 1970. · [Zbl 0215.37201](#)
- [2] S. Bosch, W. Lütkebohmert, and M. Raynaud, Néron Models , Ergeb. Math. Grenzgeb. (3), vol. 21, Springer-Verlag, Berlin, 1990. · [Zbl 0705.14001](#)
- [3] R. Hartshorne, Algebraic Geometry , Grad. Texts in Math., vol. 52, Springer-Verlag, New York, 1977. · [Zbl 0367.14001](#)
- [4] S. Lang, Algebra , 2nd ed., Addison-Wesley, Reading, Mass., 1984. · [Zbl 0712.00001](#)
- [5] S. Lattès, Sur l'iteration des substitutions rationnelles et les fonctions de Poincaré , C. R. Acad. Sci. Paris 166 (1918), 26-28. · [Zbl 46.0522.01](#)
- [6] C. McMullen, Families of rational maps and iterative root-finding algorithms , Ann. of Math. (2) 125 (1987), no. 3, 467-493. · [Zbl 0634.30028](#) · [doi:10.2307/1971408](#)
- [7] J. S. Milne, Étale Cohomology , Princeton Math. Ser., vol. 33, Princeton Univ. Press, Princeton, 1980. · [Zbl 0433.14012](#)
- [8] J. Milnor, Geometry and dynamics of quadratic rational maps , Experiment. Math. 2 (1993), no. 1, 37-83. · [Zbl 0922.58062](#) · [doi:10.1080/10586458.1993.10504267](#) · [emis:journals/EM/expmath/volumes/2/2.html](#)
- [9] P. Morton, On certain algebraic curves related to polynomial maps , Compositio Math. 103 (1996), no. 3, 319-350. · [Zbl 0860.11065](#) · [numdam:CM_1996__103_3_319_0](#) · [eudml:90474](#)
- [10] P. Morton and J. H. Silverman, Periodic points, multiplicities, and dynamical units , J. Reine Angew. Math. 461 (1995), 81-122. · [Zbl 0813.11059](#) · [doi:10.1515/crll.1995.461.81](#) · [crelle:GDZPPN002212536](#) · [eudml:153708](#)
- [11] D. Mumford and J. Fogarty, Geometric Invariant Theory , 2d ed., Ergeb. Math. Grenzgeb. (2), vol. 34, Springer-Verlag, Berlin, 1982. · [Zbl 0504.14008](#)
- [12] 1 M. Rees, A partial description of parameter space of rational maps of degree two, I , Acta Math. 168 (1992), no. 1-2, 11-87. · [Zbl 0774.58035](#) · [doi:10.1007/BF02392976](#)
- [13] 2 M. Rees, A partial description of the parameter space of rational maps of degree two, II , Proc. London Math. Soc. (3) 70 (1995), no. 3, 644-690. · [Zbl 0827.58048](#) · [doi:10.1112/plms/s3-70.3.644](#)
- [14] G. Segal, The topology of spaces of rational functions , Acta Math. 143 (1979), no. 1-2, 39-72. · [Zbl 0427.55006](#) · [doi:10.1007/BF02392088](#)
- [15] C. S. Seshadri, Geometric reductivity over arbitrary base , Advances in Math. 26 (1977), no. 3, 225-274. · [Zbl 0371.14009](#) ·

[doi:10.1016/0001-8708\(77\)90041-X](https://doi.org/10.1016/0001-8708(77)90041-X)

- [16] J. H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*, Grad. Texts in Math., vol. 151, Springer-Verlag, New York, 1994. · [Zbl 0911.14015](#)
- [17] J. H. Silverman, The field of definition for dynamical systems on \mathbb{P}^1 , *Compositio Math.* 98 (1995), no. 3, 269-304. · [Zbl 0849.11090](#) · [numdam:CM_1995__98_3_269_0](#) · [eudml:90404](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.