Ehrich, Sven

Error bounds for linear approximations on the real line. (English) Zbl 0980.41032

Analysis, München 20, No. 1, 51-63 (2000).

The author considers quasi-interpolants of the form

$$Q_h[f](x) = \sum_{k \in Z} \varphi(kh) \cdot \varphi_{k,h}(x - kh)$$

where $\varphi k, h(t) = \varphi_k(h^{-1}t)$, $|\varphi_k(t)| \leq C(1 + |t|)^{-n-\varepsilon}$, $h > 0$, $n \in \mathbb{N}$, $0 < \varepsilon \leq 1$ and C a constant independent of k and t and $Q_h[p](x) = p(x)$ for any polynomial p of degree $\leq s - 1$ (i.e. deg $(Q_h) \geq s - 1$). Let $A_{\nu,\infty}(\mathbb{R}) = \{f : \sup |f^{(\nu)}(x)| < \infty\}$, $\nu = 1, 2, ..., s$. The author develops for this class a new technique based on the Peano kernel for these interpolants and obtains new estimations of the pointwise approximation errors and new results concerning the unimprovability of these results. For instance, if $\deg(Q_h) \geq n - 1$ then for every $x \in \mathbb{R}$ there exists a constant $C > 0$, independent of h, such that for each $f \in A_{n-1,\infty}(\mathbb{R}) \cap A_{n,\infty}(\mathbb{R})$ the following inequality holds:

$$|f(x) - Q_h[f](x)| \leq C \max(\|f^{(n-1)}\|_\infty, \|f^{(a)}\|_\infty) \cdot \begin{cases} h^n \cdot |\log h|, & \varepsilon = 1 \\ h^{n-\varepsilon+1}, & 0 < \varepsilon < 1. \end{cases} .$$

Reviewer: Costica Mustăţa (Cluj-Napoca)

MSC:

41A80 Remainders in approximation formulas
65D99 Numerical approximation and computational geometry (primarily algorithms)

Keywords:

error bounds

Full Text: DOI