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Generalized gamma measures and shot-noise Cox processes. (English) Zbl 0957.60055
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For $\alpha \leq 1$, $\delta > 0$ and $\theta \geq 0$ we define the distribution $G(\alpha, \delta, \theta)$ on $[0, \infty)$ by its Laplace transform

$$L_{\alpha,\delta,\theta}(s) = \exp\left(-\frac{\delta}{\alpha}((\theta + s)^\alpha - \theta^\alpha)\right), \quad s \geq 0.$$

Reviews of this so-called $G(\alpha, \delta, \theta)$-family, which includes several important distributions on $[0, \infty)$, can be found by *P. Hougaard*, *M.-L. T. Lee* and *G. A. Whitmore* [Biometrics 53, No. 4, 1225-1238 (1997; Zbl 0911.62101)]. The fact is important that the $G(\alpha, \delta, \theta)$-measures are infinitely divisible in the parameter δ, which makes them a natural basis for a family of random measures with independent increments. Consider a Borel subset $E \subseteq \mathbb{R}^d$ equipped with its Borel σ-algebra $\mathcal{E}$ and let κ be a measure on $(E, \mathcal{E})$ which is finite on bounded sets. A random measure μ on $(E, \mathcal{E})$ is said to be a G-measure with index α, share measure κ and intensity parameter θ, if the following conditions hold: a) $\mu(A)$ follows a $G(\alpha, \kappa(A), \theta)$ distribution for every bounded $A \in \mathcal{E}$, b) μ has independent increments.

Using the Lévy representation for infinitely divisible measures, the following properties of G-measures are derived: If μ is a $G(\alpha, \kappa, \theta)$-measure with $\alpha < 1$, then

(i) μ can be represented in the form $\mu(A) = \int_0^\infty y\widetilde{N}(A \times dy)$ where $\widetilde{N}$ is a Poisson process on $E \times [0, \infty)$ with intensity measure ν given by

$$\nu(A \times B) = \frac{\kappa(A)}{\Gamma(1-\alpha)} \int_B y^{-\alpha-1} e^{-\theta y} dy.$$

Thus $\mu = \sum_i w_i \delta_{x_i}$, where δ_x is the Dirac measure at x.

(ii) μ is almost surely purely atomic.

(iii) μ has no fixed atoms if and only if κ is diffuse.

(iv) If $E = \mathbb{R}^d$, then μ is stationary if and only if κ is proportional to the Lebesgue measure on E.

G-measures are interesting in their own right as models for discontinuous random fields or considered as marked point process models. When G-measures are used as intensity measures for Cox processes, this results in completely random point processes with multiple points. While desirable for some modelling purposes this is not the case when modelling e.g. positions of trees. Thus the author studies kernel smoothed G-measures whereby diffuse measures are obtained and disjoint regions may be stochastically dependent: Let E and S be two Borel subsets of $\mathbb{R}^d$ and let $\mathcal{E}$ and $\mathcal{S}$ be the corresponding Borel σ-algebras. Consider a $G(\alpha, \kappa, \theta)$-measure $\mu = \sum_i w_i \delta_{x_i}$ on E with $\alpha < 1$ and a kernel Φ from $E \times [0, \infty)$ to S satisfying

$$\int_{E \times [0,\infty)} \varphi(A, u)\nu(du) < \infty \quad \text{for all bounded } A \in \mathcal{S}$$

where ν is defined as in (i). Then the shot-noise G-measure M with control measure μ is defined by

$$M(A) = \sum_i \Phi(A, (x_i, w_i)) \quad \text{for } A \in \mathcal{S}.$$

Several characteristics of G-measures and shot-noise G-measures are studied by simulations. Moments and mixing properties for shot-noise G-measures are derived. Finally statistical inference for shot-noise G-measures is considered and some results on nearest-neighbour Markov properties are given.

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MSC:

60G57 Random measures
60G60 Random fields
65C05 Monte Carlo methods
60G51 Processes with independent increments; Lévy processes

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Lévy representation; infinitely divisible measures; Cox processes; point processes with multiple points; G-measures; shot-noise G-measures; nearest-neighbour Markov properties

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