El-Metwally, H.; **Grove, E. A.**; **Ladas, G.**

A global convergence result with applications to periodic solutions. (English) Zbl 0971.39004

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Suppose that I is an interval of $\mathbb{R}$, that $F = F(z_1, \ldots, z_{k+1}) : I^{k+1} \to I$ is continuous, nondecreasing in each of its arguments, and that it is strictly increasing in at least two of its arguments z_i and z_j where i and j are relatively prime. Suppose further that $F(x, x, \ldots, x) = x$ for every $x \in I$. Then it is shown that every solution of

$$x_{n+1} = F(x_n, x_{n-1}, \ldots, x_{n-k}),\ n = 0, 1, \ldots$$

tends to a finite limit in I.

This result is applied to the rational recursive relation

$$x_{n+1} = \frac{A_0}{x_n} + \frac{A_1}{x_{n-2}} + \cdots + \frac{A_m}{x_{n-2m}},\ n = 0, 1, \ldots, \tag{1}$$

where m is a positive integer, $A_0, \ldots, A_m$ are nonnegative and at least two of them are positive. It is shown that if $0 \le i < j \le m$, if $2i+1$ and $2j+1$ are relatively prime, and if A_i and A_j are positive, then every positive solution of (1) converges to a period two solution.

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