Artstein, Z.; **Gaitsgory, V.**

The value function of singularly perturbed control systems. (English) Zbl 0958.49019

Appl. Math. Optimization 41, No. 3, 425-445 (2000).

In this paper the authors significantly extend certain existing results concerning the asymptotic behaviour as $\varepsilon \to 0$ of the value function

$$V_\varepsilon(t,x,y) = \inf_{u(.)} \left\{ \int_t^1 L(x(\tau), y(\tau), u(\tau)) d\tau + \psi(x(1)) \right\}, \quad (t,x,y) \in [0,1) \times \mathbb{R}^m \times \mathbb{R}^n$$

where the infimum is taken over all measurable (control) functions $u(.) : [t,1] \to U \subset \mathbb{R}^l$ that "produce" the unique (absolutely continuous) solution $(x(.), y(.))$ of the problem:

$$x'(\tau) = f(x(\tau), y(\tau), u(\tau)),\ x(t) = x,\ \varepsilon y'(\tau) = g(x(\tau), y(\tau), u(\tau)),\ y(t) = y.$$

Under some hypotheses on the value function itself one proves first that for any sequence $\varepsilon_k \to 0$ there exist a subsequence, say $\varepsilon_j \to 0$, and a "cluster function" $V(.,.)$, such that $V_{\varepsilon_j}(t,x,y) \to V(t,x)$ uniformly on compact subsets; next, the authors introduce the rather abstract "limit Hamiltonians":

$$H_0(x,\lambda) := \lim_{s\to\infty} H(x,\lambda,s,y)$$

$$H(x,\lambda,s,y) = -\inf_{u(.)} \left\{ \frac{1}{s} \int_0^s [L(x, y(\tau), u(\tau)) + \lambda f(x, y(\tau), u(\tau))] d\tau \right\}$$

where $u(.)$ are measurable control functions and $y(.)$ is the unique solution of the problem: $y'(\tau) = g(x, y(\tau), u(\tau)),\ y(0) = y$ and prove (on some 5 pages) their main result, Theorem 5.3, stating that under certain hypotheses on $V\varepsilon(.,.,.)$, $H(.,.,.,.)$, $H_0(.,.)$, any "cluster function" $V(.,.)$, of V_ε, is a viscosity solution of the ("limit") Hamilton-Jacobi equation:

$$-\frac{\partial V}{\partial t} + H_0(x, \frac{\partial V}{\partial x}) = 0,\ V(1,x) = \psi(x).$$

In Theorem 6.3 one identifies certain (more explicit) properties of the data that imply the rather implicit hypotheses of the main result and in a number of comments and examples the authors compare their results with previous work, in particular with those in [*F. Bagagiolo* and *M. Bardi*, SIAM J. Control Optimization 36, No. 6, 2040-2060 (1998; Zbl 0953.49031)] and [*P.-L. Lions*, "Generalized solutions of Hamilton-Jacobi equations" (1982; Zbl 0497.35001)], where problems "without order reduction hypothesis" are considered.

Reviewer: Stefan Mirica (Bucureşti)

MSC:

49L25 Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games
49L20 Dynamic programming in optimal control and differential games
34E15 Singular perturbations, general theory for ordinary differential equations

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