zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Englund, Erling

Perturbed renewal equations with applications to M/M queuing systems. I. (English)

Zbl 0955.60080
Theory Probab. Math. Stat. 60, 35-42 (2000) and Teor. Jmovirn. Mat. Stat. 60, 31-37 (1999).

The author considers the renewal equation

$$x_\varepsilon(t) = q(t) + \int_0^t x_\varepsilon(t-s) F_\varepsilon(ds), \quad t \geq 0, \tag{1}$$

where $q_\varepsilon(s)$ is a measurable real-valued function on $[0,\infty)$ bounded on every finite interval, $F_\varepsilon(t)$ is a distribution function on $[0,\infty)$ which is not concentrated at 0, but can be improper, i. e. $F_\varepsilon(t) \leq 1$. The author studies the exponential asymptotical relation of the following type

$$\frac{x_\varepsilon(t_\varepsilon)}{\exp\{p_\varepsilon t_\varepsilon\}} \to x_0(\infty) \quad \text{as} \quad \varepsilon \to 0.$$

For example, he finds conditions under which there exists a unique nonnegative solution of equation (1) for all ε that are small enough. He proves that for any $0 \leq t_\varepsilon \to \infty$ in such a way that $\varepsilon^z \exp\{-a/\varepsilon\} t_\varepsilon \to \lambda_z \in [0,\infty)$ for some $k \leq z \leq w$ the following asymptotical relation holds true

$$\frac{x_\varepsilon(t_\varepsilon)}{\exp\{-(b_k\varepsilon^k + \cdots + b_{z-1}\varepsilon^{z-1})e^{-a/\varepsilon} t_\varepsilon\}} \to e^{-\lambda_z b_z} x_0(\infty) \quad \text{as} \varepsilon \to 0.$$

Here b_k are some constants.

Reviewer: Yu.V.Kozachenko (Kyïv)

MSC:

60K05 Renewal theory
60K15 Markov renewal processes, semi-Markov processes
60K25 Queueing theory (aspects of probability theory)

Cited in **2** Reviews
Cited in **1** Document

Keywords:

renewal equation; balancing condition; M/M queueing system; perturbation; exponential moment

© 2021 FIZ Karlsruhe GmbH