**Hayes, Brian T.**; **LeFloch, Philippe G.**

**Nonclassical shocks and kinetic relations: Strictly hyperbolic systems.** (English)

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The authors consider a system of hyperbolic conservation laws $\partial_t u + \partial_x f(u) = 0$, $u(x,t) \in \mathcal{U}$, where $\mathcal{U}$ is a convex and open subset of $\mathbb{R}^N$, and the flux-function $f : \mathcal{U} \to \mathbb{R}^N$ is a smooth mapping. The system is endowed with a strictly convex entropy pair $(U, F)$, which implies that the system is hyperbolic. The authors study the Riemann problem in the class of solutions satisfying a single entropy inequality, the only such constraint available for general diffusive-dispersive approximations. It is shown that even in strictly hyperbolic systems, nonclassical shocks with arbitrarily small amplitudes occur. The Riemann problem can be solved uniquely using nonclassical shocks, provided an additional constraint is imposed. An admissibility criterion, called kinetic relation, by analogy with similar laws introduced in material science for propagating phase boundaries is discussed. A unique solution to the Riemann problem in the class of nonclassical solutions (when the kinetic relation is enforced) is constructed. By some interesting examples (in magnetohydrodynamics) the authors demonstrate numerically that certain diffusive-dispersive approximations generate nonclassical shocks.

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**MSC:**

35L65 Hyperbolic conservation laws
76L05 Shock waves and blast waves in fluid mechanics
35L67 Shocks and singularities for hyperbolic equations



**Keywords:**

conservation laws; hyperbolic entropy; Riemann problem; magnetohydrodynamics; diffusive-dispersive approximations; shock wave

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