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Harmonic and minimal vector fields on tangent and unit tangent bundles. (English)

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This article studies two functionals and three variational problems for unit tangent vector fields on Riemannian manifolds, the unit tangent bundle being equipped with the Sasaki metric. On a compact orientable manifold, the energy and volume functionals can be defined and their critical points examined. An important element is that while critical points of the volume are taken only for variations through unit vector fields, critical points of the energy can be chosen either among unit vector fields or among all maps from the manifold to its unit tangent bundle. This gives three different sets, minimal vector fields (for the volume), harmonic vector fields (for the energy but only among unit vector fields) and harmonic maps (for the energy among all maps into the unit tangent bundle). The last two sets are distinct.

The aim of this work is to exhibit unit vector fields which satisfy these three conditions simultaneously. The first example is on a two-point homogeneous space. For such a space, the geodesic flow vector, $\xi(x,u) = u^h$ (the horizontal lift) is minimal, harmonic and a harmonic map, and, conversely, in dimension two or three, this vector cannot be a critical point of either of the three variational problems unless the manifold has constant curvature.

The addition of a complex structure enables the authors to find, on a complex space form, new unit vector fields, namely the tangential and horizontal lifts of Ju, which are harmonic, minimal and harmonic maps. In fact these two combine to define a one-dimensional family with this property.

Finally, for the pointed tangent bundle, $TM \setminus M$, it is shown that the unit vector field, $u^v/r, (r = |u|)$ is always harmonic, minimal and a harmonic map, while u^h/r needs the manifold to be two-point homogeneous.

Reviewer: Eric Loubeau (Brest)

MSC:

53C43	Differential geometric aspects of harmonic maps
58E20	Harmonic maps, etc.
53C42	Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
53D25	Geodesic flows in symplectic geometry and contact geometry

Keywords:

minimal vector field; tangent bundle; unit tangent bundle; geodesic flow; two-point homogeneous space; harmonic vector field

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