

Conrey, J. B.; Iwaniec, H.

The cubic moment of central values of automorphic L -functions. (English) Zbl 0973.11056
Ann. Math. (2) 151, No. 3, 1175-1216 (2000).

This paper deals with cubic moments of the central values of the quadratic twists $L_f(s, \chi)$ of automorphic L -functions attached to (holomorphic or non-holomorphic) cusp forms f of a given weight k for the congruence subgroup $\Gamma_0(q)$; here q is an odd squarefree number and $\chi(n) = \left(\frac{n}{q}\right)$ is the primitive character (mod q) defined by the Jacobi symbol. First, if f runs over the primitive holomorphic cusp forms of weight $k \geq 12$ and level dividing q , then $\sum_f L_f^3(1/2, \chi) \ll q^{1+\varepsilon}$, where the implied constant depends on ε and k . Since the terms in this sum are known to be non-negative, it follows that $L_f(1/2, \chi) \ll q^{1/3+\varepsilon}$, which should be compared with the “convexity bound” with $1/2$ standing in place of $1/3$ in the exponent.

Another cubic moment considered in this paper is related to Maass cusp forms, and then there is also a contribution coming from the continuous spectrum which turns out to be the sixth moment of the Dirichlet L -function $L(1/2 + ir, \chi)$ over an interval $[-R, R]$. The estimate is as above, and as a highly remarkable corollary it follows that $L(1/2 + ir, \chi) \ll q^{1/6+\varepsilon}(|r| + 1)^A$ for some constant A ; this is the first improvement of the famous “ $3/16$ ”-estimate due to D. A. Burgess from 1962, in the case of quadratic L -functions.

The basic idea in the proofs is to translate the cubic averages into the language of Kloosterman sums by use of formulae of the Petersson or Kuznetsov type. After that, subtle and complicated arguments are still needed to work out the final results.

It should be noted in this context that another cubic moment was recently studied by A. Ivić [J. Théor. Nombres Bordeaux (to appear)]. In this case, cubes of the central values of the L -functions attached to Maass cusp forms for the full modular group were averaged over a short spectral interval, and a subconvexity estimate in the spectral aspect was deduced invoking again the non-negativity property of the L -values in question.

Reviewer: [Matti Jutila \(Turku\)](#)

MSC:

- 11F67** Special values of automorphic L -series, periods of automorphic forms, cohomology, modular symbols
- 11M06** $\zeta(s)$ and $L(s, \chi)$

Cited in **6** Reviews
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[automorphic \$L\$ -functions](#); [Dirichlet \$L\$ -functions](#); [cubic moments](#); [central values](#)

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