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Iterated Cauchy and Dirichlet problems with the Bessel operator in Banach space. (English. Russian original) Zbl 0988.34046

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Consider the operator-differential equation of order $2n > 2$,

$$(B_k - A)^n u(t) = A_0^n u(t), \quad t > 0, \tag{1}$$

in a Banach space E. Here, $A_0 \in \mathcal{B}(E)$, $B_k u(t) = u''(t) + \frac{k}{t} u'(t)$ for $t > 0$ and $k > 0$, and A satisfies the following condition: the Cauchy problem $B_k u(t) = Au(t)$, $u(0) = x_0$, $u'(0) = 0$, has a unique exponentially bounded solution.

Here, the Cauchy problem

$$\lim_{t\to 0} (B_k - A)^j u(t) = x_{j+1}, \qquad \lim_{t\to 0} ((B_k - A)^j u(t))' = 0 \tag{2}$$

for the equation (1) is investigated. As is well known, the solvability of a higher-order differential equation cannot be stated in the general case. For this special kind of equation, conditions for (1),(2) to be well posed are obtained, and a formula for the solution is given.

Moreover, the iterated Dirichlet problem

$$(B_m + A)^n w(t) = A_0^n w(t) \text{ for } t > 0,\ w_i(0) = x_i,\ \sup_{i\geq 0} |w_i(0)| \leq M, \tag{3}$$

with $w_1(t) = w(t)$, $w_i(t) = (B_m + A) w_{i-1}(t)$, $i = 2, \dots, n$, is studied. Existence and uniqueness conditions for (3) are found, and the form of the solution is given.

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Cited in **1** Document

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