Soundararajan, K.
Divisibility of class numbers of imaginary quadratic fields. (English) Zbl 1018.11054
J. Lond. Math. Soc., II. Ser. 61, No. 3, 681-690 (2000).

For any rational integer $g \geq 2$, let $\mathcal{N}_g(X)$ be the number of squarefree (positive) integer $d \leq X$ such that the ideal class group of the imaginary quadratic number field $\mathbb{Q}(\sqrt{-d})$ contains an element of order g. It is believed that $\mathcal{N}_g(X) \sim C_g X$ for some positive constant C_g, however the asymptotic formula for $\mathcal{N}_g(X)$ is still unknown except for the case $g = 2$, in which case we easily see $\mathcal{N}_2(X) \sim (6/\pi^2)X$ by genus theory. The author improves the best known result $\mathcal{N}_g(X) \gg X^{1/2+1/g}$ for general $g \geq 3$ due to *M. Ram Murty* [Topics in number theory, Kluwer Math. Appl., Dordr. 467, 229–239 (1999; Zbl 0993.11059)] to

$$\mathcal{N}_g(X) \gg X^{1/2+2/g-\varepsilon} \quad \text{if } g \equiv 0 \pmod 4$$

and

$$\mathcal{N}_g(X) \gg X^{1/2+3/(g+2)-\varepsilon} \quad \text{if } g \equiv 2 \pmod 4.$$

(Note that for odd g, we have $\mathcal{N}_g(X) \geq \mathcal{N}_{2g}(X) \gg X^{1/2+3/(2(g+1))-\varepsilon}$.) He also offers a simple proof of $\mathcal{N}_4(X) \gg X/\sqrt{\log X}$.

Reviewer: Ken Yamamura (Yokosuka)

MSC:

11R29 Class numbers, class groups, discriminants
11R11 Quadratic extensions

Keywords:

divisibility of class numbers; imaginary quadratic fields

Full Text: DOI