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Nonexistence of some antipodal distance-regular graphs of diameter four. (English)

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The authors show that for distance-regular graphs with certain intersection arrays, the first subconstituent graphs are strongly regular. Theorem 2.2. Let Γ be a nonbipartite distance-regular graph with diameter $d \geq 3$, eigenvalues $k = \theta_0 > \cdots > \theta_d$, and let $b^- = -1 - b_1/(\theta_1 + 1)$, $b^+ = -1 - b_1/(\theta_d + 1)$. Then $k(a_1 + b^+b^-) \leq (a_1 - b^+)(a_1 - b^-)$, and equality holds if and only if all local graphs are connected strongly regular graphs with eigenvalues a_1, b^+, b^-.

Let Γ be a distance-regular graph, whose local graphs are strongly regular with parameters (k', λ', μ'). Then the μ-graphs of Γ are regular with valency μ', $c_2\mu'$ is even and $c_2 \geq \mu' + 1$, with equality if and only if Γ is a Terwilliger graph (Theorem 3.1).

Corollary 3.5. Let Γ be a nonbipartite antipodal distance-regular graph with diameter four and covering index r and $k(a_1 + b^+b^-) = (a_1 - b^+)(a_1 - b^-)$. Then b^+ and b^- are integral, $b^+ \geq 1$, $b^- \leq -2$ and r divides $b^+ - b^-$.

Theorem 3.1 and Corollary 3.5 give new existence conditions for the corresponding distance-regular graphs. In particular 20 intersection arrays from tables of feasible parameters of nonbipartite antipodal distance-regular graphs with diameter 4 are ruled out.

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References:

[1] Biggs, N.L., Algebraic graph theory, (1974), Cambridge University Press Cambridge · Zbl 0501.05039
[2] A. E. Brouwer
[3] Brouwer, A.E.; Cohen, A.M.; Neumaier, A., Distance-regular graphs, (1989), Springer-Verlag Berlin, Heidelberg · Zbl 0747.05073
[4] A. E. Brouwer, A. M. Cohen, A. Neumaier
[5] Dickie, G.A.; Terwilliger, P.M., Dual bipartite \textit{Q} -polynomial distance-regular graphs, Europ. J. combinatorics, 17, 613-623, (1996) · Zbl 0921.05064
[6] Godsil, C.D., Algebraic combinatorics, (1993), Chapman and Hall New York · Zbl 0814.05075
[7] W. H. Haemers, 1979
[8] W. H. Haemers, Interlacing eigenvalues and graphs, Linear Algebr. Appl. 226, 228 · Zbl 0831.05044
[9] 1995, 593, 616
[10] Jurišić, A.; Koolen, J.; Terwilliger, P., Tight distance-regular graphs, J. algebr. comb., 12, 163-197, (2000) · Zbl 0959.05121
[11] Jurišić, A.; Koolen, J.; Terwilliger, P., Krein parameters and antipodal tight graphs with diameter 3 and 4, Discrete. math., submitted, (1999)
[12] Meixner, T., Some polar towers, Europ. J. combinatorics, 12, 397-415, (1991) · Zbl 0753.05016
[13] Soicher, L.H., Three new distance-regular graphs, Europ. J. combinatorics, 14, 501-505, (1993) · Zbl 0794.05135