Castro, Ildefonso; Urbano, Francisco

On a minimal Lagrangian submanifold of $\mathbf{C}^n$ foliated by spheres. (English) Zbl 0974.53059

Mich. Math. J. 46, No. 1, 71-82 (1999).

The Lagrangian catenoid in $\mathbb{C}^n$ is defined by $M_0 = \{(x,y) \in \mathbb{C}^n = \mathbb{R}^n \times \mathbb{R}^n : |x|y = |y|x, \operatorname{Im}(|x|+i|y|)^n = 1, |y| < |x| \tan(\pi/n)\}$. The authors prove the following interesting characterizations of the Lagrangian catenoid.

Theorem 1. Let $\phi : M \to \mathbb{C}^n$ be a minimal (nonflat) Lagrangian immersion of an n-manifold. Then M is foliated by pieces of round $(n-1)$-spheres of $\mathbb{C}^n$ if and only if ϕ is congruent (up to dilations) to an open subset of the Lagrangian catenoid.

Theorem 2. Let $\phi : M \to \mathbb{C}^m$ be a (nonflat) complex immersion of a complex n-dimensional Kähler manifold M. Then M is foliated by pieces of round $(2n-1)$-spheres of $\mathbb{C}^m$ if and only if $n = 1$ and ϕ is congruent (up to dilations) to an open subset of the Lagrangian catenoid.

Theorem 3. Let M be an n-dimensional $(n \geq 3)$ complete minimal (nonflat) submanifold with finite total scalar curvature immersed in Euclidean space $\mathbf{E}^{2n}$. Then the compactification by the inversion of M is Lagrangian for a certain orthogonal complex structure on $\mathbf{E}^{2n}$ if and only if M is (up to dilations) the Lagrangian catenoid.

Reviewer: B.-Y.Chen (East Lansing)

MSC:

53D12 Lagrangian submanifolds; Maslov index
53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)

Keywords:

Lagrangian submanifold; Lagrangian catenoid

Full Text: DOI